

**Homework problems:**

1. Convert the following grammar into Chomsky normal form:

$$\begin{aligned} S &\rightarrow AB \mid c \\ A &\rightarrow T \mid aA \\ B &\rightarrow TT \mid \varepsilon \\ T &\rightarrow bS \end{aligned}$$

2. Determine, using the CYK algorithm (“dynamic programming method”, Sipser p. 241, Lewis & Papadimitriou p. 155), whether the strings  $abba$ ,  $bbaa$  and  $bbaab$  are generated by the grammar

$$\begin{aligned} S &\rightarrow AB \mid BA \mid a \mid b \\ A &\rightarrow BA \mid a \\ B &\rightarrow AB \mid b \end{aligned}$$

In the positive cases, give also the respective parse trees.

3. Design pushdown automata recognising the following languages:

- (a)  $\{w \in \{a, b\}^* \mid w = w^R\}$ ;  
 (b) The language generated by grammar

$$S \rightarrow (S) \mid S, S \mid a$$

(Cf. Tutorial 5, Problem 3.)

**Demonstration problems:**

4. Design an algorithm for testing whether a given a context-free grammar  $G = (V, \Sigma, P, S)$ , generates a nonempty language, i.e. whether any terminal string  $x \in \Sigma^*$  can be derived from the start symbol  $S$ .
5. Design a pushdown automaton corresponding to the grammar  $G = (V, \Sigma, P, S)$ , where

$$\begin{aligned} V &= \{S, (, ), *, \cup, \emptyset, a, b\} \\ \Sigma &= \{(, ), *, \cup, \emptyset, a, b\} \\ P &= \{S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), \\ &\quad S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b\} \end{aligned}$$

6. Design a grammar corresponding to the pushdown automaton  $M = (Q, \Sigma, \Gamma, \Delta, s, F)$ , where

$$\begin{aligned} Q &= \{s, q, f\}, \quad \Sigma = \{a, b\}, \quad \Gamma = \{a, b, c\}, \quad F = \{f\}, \\ \Delta &= \{((s, e, e), (q, c)), ((q, a, c), (q, ac)), ((q, a, a), (q, aa)), \\ &\quad ((q, a, b), (q, e)), ((q, b, c), (q, bc)), ((q, b, b), (q, bb)), \\ &\quad ((q, b, a), (q, e)), ((q, e, c), (f, e))\} \end{aligned}$$