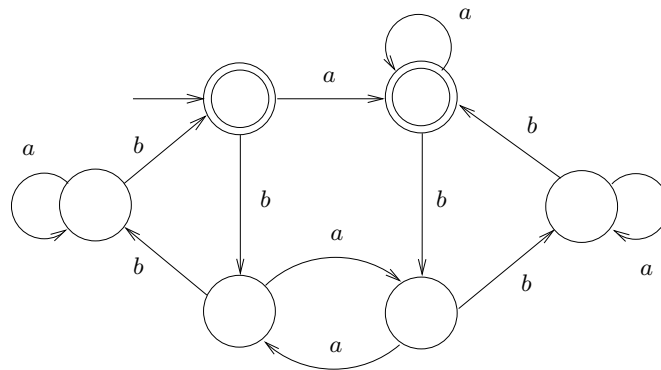


**Homework problems:**

1. Design a finite automaton that accepts precisely those binary strings that contain an even number of both 0's and 1's (e.g. 0011 and 1010, but not 001). [NB. In this and similar problems in the future, it is for simplicity always assumed that also zero is an even number, unless otherwise indicated.]
2. Design a finite automaton to control the traffic lights at an extremely low-traffic intersection. Two streets meet at the intersection, and the traffic light on each street can display either red, yellow, or green. The traffic situation is indicated by three mutually exclusive input signals: 'car arriving on street 1', 'car arriving on street 2', and 'no arriving traffic'. The automaton must ensure that each car arriving at the intersection gets to continue, and that if the traffic light in one direction is green or yellow, then the light in the intersecting direction is red. The automaton does not need to have any distinct start or final states.
3. Construct the minimal automaton corresponding to the following deterministic finite automaton:

**Demonstration problems:**

4. Formulate the model of a simple coffee machine presented in class (lecture notes p. 17) precisely according to the mathematical definition of a finite automaton (Definition 2.1). What is the formal language recognised by this automaton?
5. Show that if a language  $L \subseteq \{a, b\}^*$  is recognised by some finite automaton, then so is the language  $L^R = \{w^R \mid w \in L\}$ . (The notation  $w^R$  means the reverse of string  $w$ , cf. problem 2/2.)
6. Show that if languages  $A$  and  $B$  over the alphabet  $\Sigma = \{a, b\}$  are recognised by some finite automata, then so are the languages  $\bar{A} = \Sigma^* - A$ ,  $A \cup B$ , and  $A \cap B$ .