1. For Markov chains defined by random walks on weighted undirected connected graphs, prove that the aperiodicity condition is equivalent to the condition $\lambda_{n-1} < 2$. Give a direct prove, not appealing to the rate of convergence theorems.

2. Show that taking into account the initial distribution $f$, we can get a better bound on the rate of convergence of a random walk:

$$\| f \mathcal{P} - \pi \| \leq \exp(-s\lambda') \left( \frac{d_{\max}}{d_{\min}} \right)^{1/2} \| f \|$$

[Compare with (1.15), page 15.]

Note that $\| f \|$ can be as small as $n^{-1/2}$ for the uniform initial distribution.

3. Prove that if we modify the weights as in (1.16), page 16, the new eigenvalues are indeed $\lambda'_{k} = \lambda_{k} / (1 + c)$

4. Prove that for a random walk on a $k$-regular connected non-bipartite graph $G$ on $n$ vertices

$$\| f \mathcal{P} - \pi \| \leq \Delta(s) \cdot n^{-1/2}$$

[See the remark after Theorem 1.16, page 18.]