Problems to section 1.5 of "Spectral Graph Theory" by Fan Chung

1. For Markov chains defined by random walks on weighted undirected connected graphs, prove that the aperiodicity condition is equivalent to the condition $\lambda_{n\text{-}1} < 2$. Give a direct prove, not appealing to the rate of convergence theorems.

2. Show that taking into account the initial distribution *f*, we can get a better bound on the rate of convergence of a random walk:

$$\| fP^s - \pi \| \le \exp(-s\lambda') \, (d_{\max}/d_{\min})^{1/2} \, \| f \|$$

[Compare with (1.15), page 15.]
Note that $\| f \|$ can be as small as $n^{-1/2}$ for the uniform initial distribution.

3. Prove that if we modify the weights as in (1.16), page 16, the new eigenvalues are indeed

$$\lambda_k' = \lambda_k \, / \, (1 + c)$$

4. Prove that for a random walk on a *k*-regular connected non-bipartite graph *G* on *n* vertices

$$\| fP^s - \pi \| \le \Delta(s) \cdot n^{-1/2}$$

[See the remark after Theorem 1.16, page 18.]