1. Consider two binary linear feedback shift registers with polynomials \( f(x) = x^3 + x^2 + x + 1 \) and \( g(x) = x^4 + x + 1 \). Initialize the first register with 111, and the second one with 0101 (the registers are shifted to the left). Generate the two output sequences and take their xor-sum sequence. Determine the unique shortest linear feedback shift register that generates the sum-sequence.

2. Prove Corollary 2 of Lecture 4: If \( f(x) \) divides \( h(x) \) then \( \Omega(f) \subset \Omega(h) \).

3. Let \( e \) be the exponent of \( f(x) \). Show that then there is a sequence \( S \in \Omega(f) \) such that the period of \( S \) is equal to \( e \).

4. Determine the exponent of the polynomial \( f(x) = x^5 + x^4 + x^3 + x^2 + x + 1 \).

5. Another Fact: An irreducible polynomial \( f(x) \) is primitive if and only if \( x = 00 \ldots 010 \) is a primitive element in the field \( \mathbb{Z}_2[x]/(f(x)) \).

   We know that the polynomial \( x^4 + x^3 + x^2 + x + 1 \) is irreducible but not primitive, since its exponent is 5. Find a primitive element in the field \( \mathbb{Z}_2[x]/(x^4 + x^3 + x^2 + x + 1) \).

6. Let \( S \) be a sequence of bits with linear complexity \( L \). Its complemented sequence \( \bar{S} \) is the sequence obtained from \( S \) by complementing its bits, that is, by adding 1 modulo 2 to each bit.

   a) Show that \( LC(\bar{S}) \leq L + 1 \).

   b) Show that \( LC(\bar{S}) = L - 1 \), or \( L \), or \( L + 1 \).

7. Let us play with the set of integers \( \{0, 1, 2, \ldots, 9\} \). Given two integers from this set, generate a new number by computing the sum of the two previous numbers. If the sum is a one-digit number then the new term is equal to the sum. If the sum is a two digit number then the new term is equal to the sum of the two digits. For example, if the previous numbers are 2 and 5, then the new number is 7. And if the previous numbers are 7 and 9, the new term is 1 + 6 = 7. Describe this procedure in terms of a linear recursion over a finite ring. Show that the period of any sequence of integers generated in this manner is a divisor of 24.