T-79.5204 **Spring 2007**
Combinatorial Models and Stochastic Algorithms
Tutorial 12, April 26
Problems

This is the last set of tutorial problems. There are no more lectures in Week 17 (23.–27.4.) The course exam is scheduled for Thursday 10 May, 1–4 p.m., DCSE Lecture Hall T2. Please verify the time and place from the DCSE exam schedule, and register for the exam via the TOPI course information system. The exam will be "open book", meaning that you can take a copy of your lecture notes, together with the tutorial problem sets and their solutions to the exam.

1. Consider the following *k-Set Splitting* problem: Given a collection $\mathcal{C}$ of k-element subsets of a finite set S, is there a subset $S' \subseteq S$ such that no $C \in \mathcal{C}$ is contained in either S' or $S - S'$ (i.e., S' "splits" all the sets in $\mathcal{C}$ in two pieces). The problem is NP-complete for $k \geq 3$. Make an educated guess concerning the location of "hard instances" for this problem.

2. Consider an Ising spin system with two spins $\sigma_1, \sigma_2 \in \{-1, +1\}$ and Hamiltonian function $H(\sigma_1, \sigma_2) = -J\sigma_1\sigma_2$, $J \in \mathbb{R}$. Calculate, at given values of the coupling coefficient J and inverse temperature β, the partition function of the system, the magnetisation of each spin, and the energy of the system. (Note that the latter two quantities are expectations w.r.t. the appropriate Gibbs distribution.) Assume then that the coupling coefficient J is a random variable with density $\rho(J)$. Write down the expressions for the "quenched" averages w.r.t. J of the magnetisation and energy. What is the value of the (quenched) average ground state energy? Consider explicitly the special cases where the distibution of J is (a) uniform in an interval $[-J_0, +J_0]$, (b) normal with zero mean and unit variance.

3. [Toy "analytic continuation" example analogous to the replica method.] Consider Newton's series expansion for $(1 + x)^r$, which for integer $r \geq 0$ yields the familiar binomial formula. Show that as $r \to 0$, this leads to the series expansion in x of $\ln(1 + x)$.