Problems

1. Construct a 5-bit Hopfield associative memory network for the patterns $(+ , + , + , + , + )$, $(+ , - , - , + , - )$, and $(- , + , - , - , - )$. (See p. 67 of the lecture notes for the Hebb/Hopfield pattern storage rule.) Are the patterns stable states of the system’s dynamics? To what state does the system converge from initial state $(+ , - , + , + , + )$?

2. Derive an upper bound on the number of spin flips required for an $N$-spin SK system with integral interaction coefficients to converge to a stable state under deterministic Glauber dynamics. (Hint: Consider the amount of decrease in $H(\sigma)$ per each spin flip.) Express this bound in terms of the size $N$ and number $m$ of binary patterns stored when the system is used as a Hopfield-type associative memory. (Consider the bounds on $H(\sigma)$ that result from using the Hebb/Hopfield pattern storage rule.)

3. Compute the partition function for the binary $NK$ model where $K = 1$, and the fitness function for allele $a_i \in \{ 0 , 1 \}$ is uniformly $f^i(a_i; a_{i+1}) = a_i + Ja_{i+1} + h$, for given constants $J, h \in \mathbb{R}$. (The indexing of the loci is taken to be mod $N$.)

4. Compute the expected number of (a) edges, (b) $r$-cliques (complete subgraphs $K_r$) in a random graph $G \in G(n, p)$.

5. Derive Theorem 7.1 of the lecture notes (given any fixed graph $H$, a.e. $G \in G(n, p)$ for $0 < p < 1$ contains an induced copy of $H$) from Lemma 7.2 of the notes (for any fixed $k, l \in \mathbb{N}$, a.e. $G \in G(n, p)$ for $0 < p < 1$ has property $Q_{kl}$).