T-79.5204 **Spring 2007**
Combinatorial Models and Stochastic Algorithms
Tutorial 7, March 15
Problems

1. Construct a 5-bit Hopfield associative memory network for the patterns $(+,+,+,+,+)$, $(+,-,-,+,-)$, and $(-,+,-,-,-)$. (See p. 67 of the lecture notes for the Hebb/Hopfield pattern storage rule.) Are the patterns stable states of the system's dynamics? To what state does the system converge from initial state $(+,-,+,+,+)$?

2. Derive an upper bound on the number of spin flips required for an N-spin SK system with integral interaction coefficients to converge to a stable state under deterministic Glauber dynamics. (*Hint:* Consider the amount of decrease in $H(\sigma)$ per each spin flip.) Express this bound in terms of the size N and number m of binary patterns stored when the system is used as a Hopfield-type associative memory. (Consider the bounds on $H(\sigma)$ that result from using the Hebb/Hopfield pattern storage rule.)

3. Compute the partition function for the binary NK model where $K = 1$, and the fitness function for allele $a_i \in \{0,1\}$ is uniformly $f^i(a_i; a_{i+1}) = a_i + Ja_{i+1} + h$, for given constants $J, h \in \mathbb{R}$. (The indexing of the loci is taken to be mod N.)

4. Compute the expected number of (a) edges, (b) r-cliques (complete subgraphs K_r) in a random graph $G \in \mathcal{G}(n,p)$.

5. Derive Theorem 7.1 of the lecture notes (given any fixed graph H, a.e. $G \in \mathcal{G}(n,p)$ for $0 < p < 1$ contains an induced copy of H) from Lemma 7.2 of the notes (for any fixed $k, l \in \mathbb{N}$, a.e. $G \in \mathcal{G}(n,p)$ for $0 < p < 1$ has property Q_{kl}).