T-79.5204 **Spring 2007**
Combinatorial Models and Stochastic Algorithms
Tutorial 4, February 15
Problems

1. Apply the method of canonical paths to the random walk described in Problem 5 of last week's tutorial. Thus, the task is to calculate using this method an upper bound on the mixing time of a simple symmetric random walk on an $n \times n$ square lattice with self-loop parameter $0 < 1 - \beta < 1$ and periodic boundary conditions (i.e. each node (i, j), $i, j = 0, \ldots, n-1$, has as neighbours the nodes $(i \pm 1, j \pm 1) \bmod n$).

2. Calculate, using the method of canonical paths, an upper bound on the mixing time of the simple random walk on a Boolean hypercube $B_n = \{0, 1\}^n$, where at each node u there is a self-loop probability of $1/2$, and otherwise a uniform probability $1/2n$ of moving to any of the n nodes at Hamming distance 1 from u.

3. [**Bonus problem.**] The techniques presented in Examples 3.1 and 3.2 in the lecture notes yield for the mixing time of a simple random walk on a cycle of n nodes a lower bound of $\Omega(n \cdot \ln \frac{1}{\varepsilon})$ and an upper bound of $O(n^2 \ln n \cdot \ln \frac{1}{\varepsilon})$. Determine, by whatever means, which of these bounds is closer to being tight.