T-79.5202 Combinatorial algorithms – home assignment 1/08

How many different 4-bit binary Gray codes are there?

We represent a Gray code as an ordered list of codewords such that the first codeword is always 0000, and consecutive codewords differ at exactly one position. Gray codes are considered different, if the ordered lists are different.

- How many different 4-bit binary Gray codes are there?
- A Gray code is cyclic, if its first and last codeword differ at exactly one position. How many of the 4-bit Gray codes are cyclic?
- What is the first 4-bit Gray code in lexicographical order?

This assignment is most likely too laborious to be solved by hand. A submitted answer must consist of a sufficiently detailed description of the solution method and results obtained (in good-quality PDF), and the source code of the computer programs designed (if any) – see separate instructions on the course WWW page. Some suggestions / hints: How could rank and unrank functions be used? Could the problem be interpreted geometrically?

Insightful observations, additional experimentation (such as considering the 5-bit case), or algorithmic ideas are considered extra merit; the following suggested extra credit problems are already somewhat challenging.

- The delta sequence of a cyclic Gray code is the ordered list $[\delta_1, \delta_2, \ldots, \delta_{2^n}]$, where δ_i denotes the position where the ith and $i+1$th $\pmod{2^n}$ codeword differ. Two cyclic Gray codes are equivalent, if their delta sequences are equivalent. Two delta sequences are equivalent, if one can be obtained from the other by rotation $[\delta_1, \ldots, \delta_{2^n}] \to [\delta_i, \delta_{i+1}, \ldots, \delta_{2^n}, \delta_1, \ldots, \delta_{i-1}]$, mirroring $[\delta_1, \ldots, \delta_{2^n}] \to [\delta_{2^n}, \delta_{2^n-1}, \ldots, \delta_1]$, permuting the coordinates $[\delta_1, \ldots, \delta_{2^n}] \to [\pi(\delta_1), \ldots, \pi(\delta_{2^n})]$, where π permutes the set $\{1, 2, 3, 4\}$, or combining these operations. Into how many equivalence classes can the delta sequences (and thus also cyclic Gray codes) be partitioned and how many codes are there in each equivalence class?
- Could the answer to the previous questions be determined without having to catalogue all cyclic Gray codes first?
- Could one obtain, if not the exact value, at least some lower bound for the number of 6-bit Gray codes (cyclic and/or non-cyclic)?

Other interesting aspects of these structures may also be considered.

Submit your answer to the teaching assistant (Petri Savola, E-mail: pjsavola©tcs.hut.fi) no later than on Friday 15 February. Follow the instructions on the course WWW page. Note that points will be deducted for submitting late.