Home assignment 2

Banknotes and coins in the common currency of the European Union, the euro, came into use in 2002. In addition to the cent-valued coins, the 1 and 2 euro coins and the 5, 10, 20, 50, 100, 200 and 500 euro banknotes came into use.

One possible criterion for evaluating a selection of monetary denominations[1] is to consider what kind of sums can be paid so that only a small number of monetary tokens (coins or banknotes) changes hands. For example, 46 euro could be paid by giving a 50 euro banknote and receiving two 2 euro coins as change. It could also be paid by giving a 50 euro banknote and a 1 euro coin and receiving a 5 euro banknote as change.

Let $L(k, m)$ be the largest integer L for which it is possible to find a set of denominations that consists of k integers such that every possible sum of money $1, \ldots, L$ can be paid so that at most m monetary tokens change hands. In other words, $L(k, m)$ is the largest integer L for which there exists a k-element set $W = \{w_1, \ldots, w_k\} \subset \mathbb{N}$ such that for all $x \in \{1, \ldots, L\}$ there exist integer coefficients $m_1, \ldots, m_k$ for which $x = \sum_{i=1}^{k} m_i w_i$ and $\sum_{i=1}^{k} |m_i| \leq m$.

- What is the smallest (integer) sum of money that cannot be paid with the set of denominations $\{1, 2, 5, 10, 20, 50, 100, 200, 500\}$ so that at most three monetary tokens change hands?

- What kind of a set of denominations should the European Union have taken into use? Design a computer program based on a nontrivial search heuristic (simulated annealing, tabu search, genetic algorithm, or similar) for searching for a good lower bound for $L(k, 3)$, where $7 \leq k \leq 9$. You may test your program on how quickly it finds sets of denominations corresponding to the known values $L(1, 3) = 3$, $L(2, 3) = 12$, $L(3, 3) = 24$, $L(4, 3) = 47$, $L(5, 3) = 73$, $L(6, 3) = 110$.

Evaluation criteria: a report can be considered especially good, if for example the solutions obtained are particularly good compared to the computation time expended, or an interesting comparison between various search strategies is presented.

Reports should be submitted according to the instructions on the course WWW page to `Harri.Haanpaa@hut.fi` no later than Wednesday 15 March, 2006. In particular, remember to give the best lower bounds for $L(k, m)$ you were able to find along with the corresponding sets of denominations, and to report the computation time expended.

[1] in Finnish: nimellisarvo