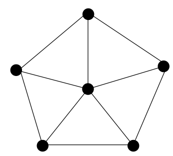
T-79.5201 Discrete Structures, Autumn 2007

Tutorial 6, 14 November

1. A k-*wheel* is a graph that consists of a $(k-1)$-cycle of nodes, each connected by an edge ("spoke") to a central node (the "hub"). Thus, the following is a 6-wheel:

   Prove that the graph property "G contains a k-wheel" has a threshold function for any fixed $k \geq 4$, and compute it.

2. Prove that the graph property "G contains a connected subgraph of at least k nodes" has a threshold function for any fixed $k \geq 2$, and compute it.

3. By a result of Komlós and Szemerédi (1983), the threshold function for a random graph to be Hamiltonian, i.e. to contain a Hamiltonian cycle, is essentially $(\ln n + \ln \ln n)/n$. (More precisely, for any function $\omega(n) \to \infty$, $(\ln n + \ln \ln n - \omega(n))/n$ is a lower threshold and $(\ln n + \ln \ln n + \omega(n))/n$ is an upper threshold.)

   Derive from this result the following proposition for *directed* random graphs: there is a constant c such that if $p = c(\ln n/n)^{1/2}$, then a.e. directed graph contains a directed Hamiltonian cycle. (*Hint:* What is the probability that for a given pair of vertices u and v, a random directed graph contains both edges (u, v) and (v, u)? Apply the Komlós-Szemerédi bound to the random undirected graph formed by the double edges.)