# T-79.5201 Discrete Structures, Autumn 2007

Tutorial 3, 10 October

1. [Alon & Spencer, Prob. 3.1:] Based on the fact (Thm 3.1 in the lecture notes, Thm 3.1.1. in Alon & Spencer) that the Ramsey number $R(k)$ satisfies, for every integer $n$,

$$R(k) > n - \binom{n}{k} 2^{1-\binom{k}{2}},$$

conclude that

$$R(k) \geq (1 - o(1)) \frac{k}{e} 2^{k/2}.$$

2. [Alon & Spencer, Prob. 3.2:] Consider the off-diagonal Ramsey numbers $R(k, t)$. Prove that for any integer $n$ and $p \in [0, 1]$,

$$R(k, t) > n - \binom{n}{k} p^{\binom{k}{2}} - \binom{n}{t} (1 - p)^{\binom{t}{2}}.$$

Conclude from this that

$$R(4, t) = \Omega((t/\ln t)^2).$$

3. [Alon & Spencer, Prob. 3.3:] A *k-uniform hypergraph* on a vertex set $V$ is a family of subsets ("hyperedges") of $V$, each of size $k$. (Thus, a usual graph is a "2-uniform hypergraph".) Thm 3.4 in the lecture notes (Thm 3.2.1 in Alon & Spencer) gives a lower bound on the independence number of 2-uniform hypergraphs. Prove that every 3-uniform hypergraph with $n$ vertices and $m \geq n/3$ edges contains an independent set of size at least

$$\frac{2n\sqrt{n}}{3\sqrt{3m}}.$$