T-79.5201 Discrete Structures, Autumn 2007
Tutorial 1, 26 September

1. (a) Let $R(k)$ be the kth Ramsey number, as defined in the lectures. Verify that $R(3) = 6$.
   (b) Denote by $T(k)$ the kth "tournament number", as defined in the lectures. Verify that $T(1) = 3$ and $T(2) = 7$.

2. Verify the estimates $\binom{n}{k} < \left(\frac{en}{k}\right)^k$ and $(1-p)^n < e^{-np}$ for $0 < p < 1$. Using these estimates, verify the bound

$$T(k) \leq k^2 \cdot 2^k (\ln 2) \cdot (1 + o(1)).$$

   on the size of the kth "tournament number". (*Note:* The given upper bound on the sizes of binomial coefficients is not quite obvious. You may wish to consult the literature for the clever short derivation.)

3. [Alon & Spencer, Prob. 1.10:] Prove that there is an absolute constant $c > 0$ with the following property. Let A be an n by n matrix with pairwise distinct entries. Then there is a permutation of the rows of A so that no column in the permuted matrix contains an increasing subsequence of length at least $c\sqrt{n}$.