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T-79.5201 Discrete Structures (4 cr)
Exam 21 Dec 2007, 1–4 p.m.

Write down on each answer sheet:
- Your name, department, and student number
- The text: "T-79.5201 Discrete Structures 21.12.2007"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

USE OF LECTURE NOTES, SOLUTIONS TO TUTORIAL PROBLEMS, AND ANY HANDBOOK OF MATHEMATICAL FORMULAS PERMITTED. PROGRAMMABLE AND SYMBOLIC ALGEBRA CALCULATORS FORBIDDEN.

1. Prove that the graph property "G has minimal degree at least d" (i.e. G contains a vertex of degree at least d) has a threshold function for any fixed $d \geq 1$, and compute it. *7p.*

2. Recall that an *independent set* in a graph $G = (V,E)$ is a set of vertices $U \subseteq V$ no two of which are neighbours, i.e. if $u,v \in U$, then $\{u,v\} \notin E$. Show that if G is a graph on the vertex set $[n] = \{1,\ldots,n\}$, such that the degree of vertex i is d_i, then G contains an independent set of size at least
$$\sum_{i=1}^{n} \frac{1}{d_i+1}.$$
(*Hint:* Consider a random permutation σ of the vertices, and make up a rule for including a vertex i into the independent set U, based on its relative position in σ.) *8p.*

3. Assume that n pairs of users need to communicate using edge-disjoint paths on a given network. Each pair $i = 1,\ldots,n$ can choose a path from a collection F_i of m paths. Prove that it is always possible to choose the desired n edge-disjoint communication paths, provided that for any two path sets F_i and F_j, no path in F_i intersects (shares edges with) more that $k = m/8n$ paths in F_j. (*Hint:* Consider the probability space defined by each pair of users i choosing a communication path uniformly at random from the respective set F_i, and the family of "bad" events E_{ij} of two chosen paths intersecting.) *7p.*

4. Design an efficient deterministic algorithm for actually *finding* an independent set satisfying the size lower bound given in problem 2, from a given input graph G. *8p.*

Total 30p.