1. Use Darboux’s lemma (Theorems 7.2 & 7.3 in the lecture notes) to estimate the coefficients of the following generating functions:

   (a) \( f(z) = e^{-z/2} \sqrt{1 - z} \),

   (b) \( f(z) = e^{-z+z^2/2} \sqrt{1 - z^2} \).

2. The exponential generating function for the class of involutions is \( \hat{t}(z) = e^{z+z^2/2} \). (Cf. e.g. p. 28 of the lecture notes.) Use this fact to estimate the number \( t_n \) of involutions of \( n \) elements.

3. The exponential generating function of the Bell numbers, i.e. the numbers of partitions \( b_n \) of \( n \)-element sets is \( \hat{b}(z) = \exp(e^z - 1) \). (Cf. e.g. tutorial 3 problem 2, or p. 25 of the lecture notes.) Use this fact to estimate the size of the numbers \( b_n \).

In case you want to investigate the quality of your estimates in problems 2 and 3, you can easily obtain initial segments of the respective sequences from the “Online Encyclopedia of Integer Sequences” server, http://www.research.att.com/~njas/sequences/. Alternatively, you can determine the recurrence formulas for computing the exact values by applying the “\( zD \log \) trick” to the egf’s given in the problems, or recall them from previous exercises.