T-79.515 Cryptography: Special Topics
Some ID-PKC signcryption schemes

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1 Introduction

The traditional method for transmitting sensitive information over untrusted networks has been to first sign it with a cryptographic key and encrypt it with another cryptographic key. These traditional sign-then-encrypt schemes are composed of a separate signature scheme and an encryption scheme such that the encryption scheme could be applied without the signature scheme.

A signcryption scheme, originally introduced by Zheng in [3], combines these two operation tightly, thereby achieving improved efficiency [3]. An informal way to describe a signcryption way is to say that it is a cryptographic protocol which digitally signs messages and then encrypts them in such a manner, that the signing and encryption operations are not totally disjoint.

Identity based public key cryptography (ID-PKC) was originally proposed by Shamir in 1984 [4]. The idea is that the public key corresponding to an identity can be derived from a canonical representation of that identity in a deterministic manner. This requires that there must exist some additional secret information which provides the randomness in the key. A KGC (Key Generation Center) is used to create the "public parameters" of the system and the secret keys for each identity. ID-PKC was surveyed in [5].

This survey presents the basic security notions for identity-based signcryption schemes (signcryption schemes based on ID-PKC) and surveys a set of signcryption schemes and their security properties.

2 A Simple Example

A signcryption scheme can be constructed from any non-interactive one-round authenticated key-agreement protocol. Such a construction will lack the forward secrecy and non-repudiation properties. The protocol by Zheng [3] is presented here is a nice example, although it is not an identity-based scheme. The security of the Zheng algorithm is treated rigorously in [8].

A set of global parameter must be generated by a trusted party. These are a pair of large primes p, q and a generator g s.t. $q|p-1$, ord $g \in Z_p^* = q$ and the difficulty of solving the decisional Diffie-Hellman problem in $< g > \subseteq Z_p^*$ is exponential in the security parameter k. Let h be a hash function with range at least 2^{2k}. Let **mac** be a keyed hash function. Let (E, D) be the encryption and decryption functions of a symmetric encryption scheme.

A private key is an integer $x \in Z_q^*$ and the corresponding public key $y = g^x \bmod p$. Denote the keys of A as x_A and y_A.

The **Signcrypt**(m, x_A, B) operation which signs and encrypts a message from A to B proceeds as follows:

- A chooses randomly $r \in Z_q^*$.
- $k = h(y_B^x \bmod p)$. Split k into k_1, k_2 s.t. $k = k_1||k_2$ and k_1 is a key for (E, D) and k_2 is a key for kh.
- $r = \mathbf{mac}_{k_2}(m)$.
- $s = x(r + x_A)^{-1} \bmod q$
- $C = E_{k_1}(m)$.
- $\varphi = (C, r, s)$

The **Unsigncrypt**(m, x_B, A) operation which validates and decrypts a message from A to B proceeds as follows:

- $k = h((y_A g^r)^{s x_b} \bmod p)$
- Split k into k_1 and k_2 as above.
- m = $D_{k_1}(C)$
- Accept if $r = \mathbf{mac}_{k_2}(m)$.

Note that $(y_A g^r)^{s x_b} = g^{(x_A + rx)/x(x_A + r) * x_b} = g^{x_b x} = y_B^x$.

3 Security Model

Definition 3.1 The protocols **Setup**, **Keygen**, **Signcrypt** and **Unsigncrypt** are said to be an identity based signcryption scheme (ID-SC) if the following conditions hold:

- **Setup**(k). The setup algorithm computes a set of public parameters *params* upon input k (the security parameter).
- **Keygen**(ID). The keygen algorithm computes private key d_{ID} upon input ID (canonical representation of identity ID) and *params*.
- **Signcrypt**(m, d_A, B). The signcryption algorithm creates an encrypted and signed representation (φ) of message m upon input d_A (secret key of signer), B (canonical representation of identity B), m and *params*.
- **Unsigncrypt**(φ, d_B, A). The unsigncryption algorithm decrypts (producing m) and verifies the signature in the ciphertext φ given as input the ciphertext (φ), the secret key of the recipient (d_B), the canonical representation of identity of the signer (A) and *params*.

The (deterministic) public key derivation algorithms are interpreted be embedded in the signcryption and unsigncryption operations in this definition.

The definition above is based on the one presented in [2]. It was re-iterated here to give a framework for presenting the security notions.

Definition 3.2 Let (**Setup**, **Keygen**, **Signcrypt** and **Unsigncrypt**) be a signcryption scheme. Let $\mathcal{A}$ be an adversary which has access to a **Signcrypt** oracle (given two identities and a message, outputs the signed ciphertext φ), **Keygen** oracle (given an identity ID outputs d_{ID}), an **Unsigncrypt** oracle (given two identities and a signed ciphertext outputs the validity of the signature and the message m) and the public parameters *params*.

The scheme is said to be indistinguishable against adaptive chosen ciphertext attacks if $\mathcal{A}$ after a polynomial number of oracle queries cannot generate a pair of messages (M_0, M_1) and identities (A, B) such that when challenged with a ciphertext $C =$ **Signcrypt**(M_b, d_A, B), $\mathcal{A}$s advantage in guessing b correctly after another polynomial number of oracle queries is negligible (ie. if the guess of $\mathcal{A}$ is b' then (2Pr[b' = b] - 1) is negligible). The adversary $\mathcal{A}$ is not allowed to query the oracle for the secret keys of A or B or the unsigncrypt oracle for C at any time.

The above property requires in essence that a polynomial amount of valid keypairs, valid (message, ciphertext, keypairs) triplets do not help in deriving any information from a correctly signed ciphertext, unless the secret key is available, or the plaintext is known a priori.

Definition 3.3 Let (**Setup**, **Keygen**, **Signcrypt** and **Unsigncrypt**) be a signcryption scheme. The adversary $\mathcal{A}$ has again access to the same oracles as in the previous definition.

The scheme is said to be secure against existential forgery for adaptive chosen message attacks if a polynomially bounded adversary $\mathcal{A}$ cannot after a polynomial number of oracle queries with non-negligible probability generate a triple (φ', A, B) such that **Unsigncrypt**(φ, d_A, B) accepts the signature on φ'. The adversary is not allowed to query the secret key of B from an oracle.

!Please note that [2] allows the adversary to compute the private key of the signer and the public key of the recipient. I believe this to be a typo, and have hence corrected the definition, which now only allows the adversary to hold the decryption key, but not the signing key.

Definition 3.4 Let G_1 and G_2 be groups of prime order q, $e : G_1 \times G_1 \to G_2$ be a bi-

linear map and P be a generator of G_1. G_1 is an additive group and G_2 is a multiplicative group. The *Bilinear Diffie-Hellman problem* (BDHP) in (G_1, G_2, e) is to compute $e(P,P)^{abc}$ given (P, aP, bP, cP). The *Decisional Bilinear Diffie-Hellman problem* (DBDH(G_1, P)) is to decide whether $h = e(P,P)^{abc}$ given (P, aP, bP, cP) and $h \in G_2$.

4 Malone-Lee signcryption scheme

The Malone-Lee signcryption scheme is a signcryption scheme upon which both the Nalla-Reddy and Libert-Quisquater schemes are partly based. This scheme is presented here briefly for completeness.

The **Setup**(k) randomized algorithm is run by the KGC and given the security parameter k it generates the parameters:

- An additive group G_1 of prime order q
- A multiplicative group G_2 of prime order q (same as above).
- A generator P of G_1.
- A bilinear map $e : G_1 \times G_1 \rightarrow G_2$ such that the Bilinear Diffie-Hellman Problem in (G_1, G_2, e) is a hard problem.
- A master key $s \in F_q^*$
- A public parameter $P_{pub} = sP$.
- A hash function $H_1 : \{0,1\}^* \rightarrow G_1$.
- A hash function $H_2 : \{0,1\}^* \rightarrow F_q^*$.
- A hash function $H_3 : G_2 \rightarrow \{0,1\}^n$.

All parameters except s are public.

The **Keygen**(ID) which computes the key-pair given an identity ID is a simple hash $Q_{ID} = H_1(ID)$ of the canonical representation of the identity. The private key is $d_{ID} = sH_1(ID)$.

The **Signcrypt**(m, d_A, B) proceeds as follows:

- Let $Q_A = H_1(A)$.
- Let $Q_B = H_1(B)$.
- Randomly choose $x \in F_q^*$.
- Let $U = xP$.
- Let $r = H_2(U||m) \in F_q^*$.
- Let $c = m \oplus H_3(e(P_{pub}, Q_B)^x)$.
- Let $V = xP_{pub} + rd_A \in G_1$.
- Output the signed ciphertext as $\varphi = (U, V, c)$.

The **Unsigncrypt**$(\varphi = (U, V, c), d_A, B)$ proceeds as follows:

- Let $Q_A = H_1(A)$.
- Let $Q_B = H_1(B)$.
- Let $y = e(U, d_B)$
- Let $m = H_3(y) \oplus c$.
- Let $r = H_2(U||m)$.
- Accept the signature iff $e(V,P) = e(Q_A, P_{pub})^r e(U, P_{pub})$.

The consistency of the scheme is easy to verify based on the properties of bilinear maps. Examples of the algebraic manipulations used are shown in the following sections.

5 Modifying Zhengs Scheme for ID-SC

Nalla and Reddy have used the principles inherent in Zhengs scheme and constructed a similar ID-SC scheme. They acknowledge that they do not yet know whether the scheme is secure or not, but are working on a formal security proof. The scheme is defined in terms of the algorithms **Setup**, **Keygen**, **Signcrypt** and **Unsigncrypt**.

The **Setup**(k) randomized algorithm is run by the KGC and given the security parameter k it generates the parameters:

- An additive group G_1 of prime order q
- A multiplicative group G_2 of prime order q (same as above).
- A generator P of G_1.
- A bilinear map $e : G_1 \times G_1 \rightarrow G_2$ such that the Bilinear Diffie-Hellman Problem in (G_1, G_2, e) is a hard problem.

- A master key $s \in F_q^*$
- A public parameter $P_{pub} = sP$.
- A hash function $H_1 : \{0,1\}^* \rightarrow G_1$.
- A hash function $H_2 : \{0,1\}^* \rightarrow Z_q^*$.
- A hash function $H_3 : Z_q^* \rightarrow \{0,1\}^*$.

All parameters except s are public.

The **Keygen**(ID) which computes the key-pair given an identity ID is a simple hash $Q_{ID} = H_1(ID)$ of the canonical representation of the identity. The private key is $d_{ID} = sH_1(ID)$.

The **Signcrypt**(m, d_A, B) proceeds as follows:

- Let $Q_A = H_1(A)$.
- Let $Q_B = H_1(B)$.
- Randomly choose $x \in Z_q^*$.
- Compute $R = xQ_B \in G_1$.
- Compute $S = xP + H_2(m)Q_A \in G_1$.
- Let $k_1 = H_3(e(R, P_{pub})e(Q_B, H_2(m)d_A))$
- Let $C = k_1 \oplus m$.
- Output the signed ciphertext as $\varphi = (R, S, C)$.

The **Unsigncrypt**$(\varphi = (R, S, C), d_A, B)$ proceeds as follows:

- Let $Q_A = H_1(A)$.
- Let $Q_B = H_1(B)$.
- Let $k_2 = H_3(e(d_B, S))$.
- Let $m = k_2 \oplus C$.
- Accept the signature iff $e(R, P_{pub})e(d_B, Q_A)^{H_2(m)} = e(d_B, S)$.

Note that

$$e(R, P_{pub})e(d_B, Q_A)^{H_2(m)} = e(xQ_B, sP)e(sQ_B, H_2(m)Q_A) = e(sQ_B, xP)e(sQ_B, H_2(m)Q_A) = e(sQ_B, xP + H_2(m)Q_A) = e(d_B, S).$$

The scheme is defined for constant-sized blocks, as it uses k_1 as a pseudo-random key stream used to pad the plaintext. A trivial extension would be to use a symmetric cipher and use k_1 as a key get around this limitation.

6 A Secure Identity Based Signcryption Scheme

Libert and Quisquater propose a secure ID-SC scheme in [2]. In this version I will refer to this scheme as the Libert-Quisquater scheme. The elementary (un-optimized) version of the scheme is presented here. The scheme is an improvement on the Malone-Lee signcryption scheme [6], but is quite similar to the scheme by Nalla and Reddy shown above. The Libert-Quisquater scheme consists of the algorithms **Setup**, **Keygen**, **Signcrypt** and **Unsigncrypt**.

The **Setup**(k) randomized algorithm is run by the KGC and given the security parameter k it generates the parameters:

- An additive group G_1 of prime order q
- A multiplicative group G_2 of prime order q (same as above).
- A generator P of G_1.
- A bilinear map $e : G_1 \times G_1 \rightarrow G_2$.
- A master key $s \in F_q^*$
- A public parameter $P_{pub} = sP$.
- A hash function $H_1 : \{0,1\}^* \rightarrow G_1$.
- A hash function $H_2 : G_2 \rightarrow \{0,1\}^n$.
- A hash function $H_3 : \{0,1\}^* \times G_2 \rightarrow F_q$.
- A symmetric cryptosystem (E, D).

All parameters except s are public.

The **Keygen**(ID) which computes the key-pair given an identity ID is simple. The public key is $Q_A = H_1(A)$ and the private key $d_A = sQ_A = sH_1(A)$ (note that G_1 is an additive group).

The **Signcrypt**(m, d_A, B) algorithm proceeds as follows:

- Let $Q_B = H_1(B) \in G_1$.
- Randomly choose $x \in F_q^*$.
- Let $k_1 = e(P, P_{pub})^x$.
- Let $k_2 = H_2(e(P_{pub}, Q_B)^x)$.
- Compute the ciphertext $c = E_{k_2}(m)$.

- Let $r = H_3(c, k_1)$
- Let $S = xP_{pub} - rd_A \in G_1$.
- Output (c, r, S) as the ciphertext φ.

The **Unsigncrypt**$(\varphi = (c, r, S), d_A, B)$ proceeds as follows:

- Let $Q_A = H_1(A) \in G_1$.
- Let $k_1 = e(P, S)e(P_{pub}, Q_A)^r$.
- Let $\tau = e(S, Q_B)e(Q_A, d_B)^r$.
- Let $k_2 = H_2(\tau)$.
- Decrypt the plaintext $m = D_{k_2}(c)$.
- Accept iff $r = H_3(c, k_1)$.

Note that the following equations hold due to bilinearity.

$$e(P, S)e(P_{pub}, Q_A)^r = e(P, xP_{pub} - rd_A)e(rP_{pub}, rQ_A) = e(P, xP_{pub})e(P, rd_A)^{-1}e(P_{pub}, rQ_A) = e(P, xP_{pub})e(P, rd_A)^{-1}e(P, srQ_A) = e(P, P_{pub})^x.$$

$$e(S, Q_B)e(Q_A, d_B)^r = e(xP_{pub} - rd_A, Q_B)e(Q_A, d_B)^r = e(xP_{pub}, Q_B)e(rsQ_A, Q_B)^{-1}e(Q_A, sQ_B)^r = e(P_{pub}, Q_B)^x e(Q_A, Q_B)^{-rs}e(Q_A, Q_B)^{rs} = e(P_{pub}, Q_B)^x.$$

Theorem 6.1 *In the random oracle model, let $\mathcal{A}$ be an adversary attempting to distinguish messages using a chosen ciphertext attack, as described in Definition 1. If $\mathcal{A}$ is able to distinguish the encryptions of messages M_0 and M_1 with advantage ε in running time t and performing at most q_{H_1} H_1 operations, at most q_R H_3 operations, at most q_r* **Signcrypt** *operations and q_U* **Unsigncrypt** *operations, then there exists an algorithm $\mathcal{B}$ that can solve the Decisional Bilinear Diffie-Hellman problem in time $\mathcal{O}(t + (8q_R^2 + 4q_U)T_\varepsilon)$ with advantage $Adv(\mathcal{B})^{DBDH(G_1,P)} > 2(\varepsilon - q_U/2^{k-1})/q_{H_1}^4$ where T_ε denotes the computation time of the bilinear map.*

This theorem states that the Libert-Quisquater scheme is a secure encryption scheme in the random oracle model. A proof of this theorem is provided in appendix A of [2], but is omitted here. The proof depends on H_1, H_2 and H_3 being random oracles, which are used to create instances of the decisional bilinear Diffie-Hellman problem. In the proof pairs of tuples $c_b =$ **Signcrypt**(m_b, d_A, Q_B) are created and provided to an oracle acting as a distinguisher for chosen ciphertexts. The proof (or the scheme) provide no explicit requirements for E, but for the validity of the proof it is sufficient that E when keyed with the output of a random oracle is a random permutation. It seems that a weaker condition would at least affect the boundary of the advantage of $\mathcal{B}$.

Libert and Quisquater also argue that the scheme is secure against existential forgery for adaptive chosen message attacks. They do not provide an explicit proof, but state that a reduction exists to a variant of a scheme by Hess which has been proven secure against existential forgery for adaptive chosen message attacks.

7 Conclusion

This survey presented a set of signcryption schemes. One due to Zheng, one due to Nalla and Reddy and one due to Libert-Quisquater. The Malone-Lee scheme which is the inspiration for both the Libert-Quisquater and Nalla-Reddy schemes was also sketched. The skeleton of all schemes is highly similar, although their security properties differ. Based on the Libert-Quisquater work one can state that efficient and secure Identity Based Signcryption schemes exist.

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