# A Glimpse Beyond

- $\mathbf{EXP} = \mathbf{TIME}(2^{n^k})$
- $\mathbf{NEXP} = \mathbf{NTIME}(2^{n^k})$
- If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{EXP} = \mathbf{NEXP}$.
- If $f(n)$ and $g(n) \geq n$ are proper complexity functions, then $\mathbf{TIME}(f(n)) = \mathbf{NTIME}(f(n))$ implies $\mathbf{TIME}(g(f(n))) = \mathbf{NTIME}(g(f(n)))$.
- $\mathbf{EXP} = \mathbf{APSPACE}$
- $\mathbf{EXP}/\mathbf{NEXP}$ provably intractable (not in $\mathbf{P}$)!

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## Succinct Problems

- A type of $\mathbf{EXP}/\mathbf{NEXP}$-complete problems: $\mathbf{EXP}$ and $\mathbf{NEXP}$ are $\mathbf{P}$ and $\mathbf{NP}$ but on *exponentially more succinct* input.
- A succinct representation of a graph with $n$ nodes where $n = 2^b$ as a Boolean circuit $C$:
  $C$ has $2b$ input gates and its value on two $b$ bit integers $i, j$ is **true** iff there is a edge $[i, j]$ in the graph.
- SUCCINCT HAMILTON PATH: Given a succinct representation $C$ of a graph $G_C$, does $G_C$ have a Hamilton path?
- SUCCINCT CIRCUIT SAT, SUCCINCT 3SAT and SUCCINCT HAMILTON PATH are $\mathbf{NEXP}$-complete.
- SUCCINCT CIRCUIT VALUE is $\mathbf{EXP}$-complete.

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## First-Order Satisfiability

- The general case undecidable
- Some decidable cases of first-order logic (no function symbols or equality):
  - Schönfinkel-Bernays class: $\exists x_1 \ldots \exists x_k \forall y_1 \ldots \forall y_l \phi$ ($\mathbf{NEXP}$-complete)
  - Ackermann class: $\exists x_1 \ldots \exists x_k \forall y \exists x_{k+1} \ldots \exists x_l \phi$ ($\mathbf{EXP}$-complete)
  - Gödel class: $\exists x_1 \ldots \exists x_k \forall y_1 \forall y_2 \exists x_{k+1} \ldots \exists x_l \phi$ ($\mathbf{NEXP}$-complete)
  - Monadic class ($\mathbf{NEXP}$-complete)

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## Beyond NEXP

- $\mathbf{EXPSPACE} = \mathbf{SPACE}(2^{n^k})$
- 2-$\mathbf{EXP} = \mathbf{TIME}(2^{2^{n^k}})$
  2-$\mathbf{NEXP} = \mathbf{NTIME}(2^{2^{n^k}})$
- 3-$\mathbf{EXP} = \mathbf{TIME}(2^{2^{2^{n^k}}})$
  ...
- $\mathbf{ELEMENTARY} = \mathbf{TIME}(2^{2^{2^{\cdot^{\cdot^{\cdot^{2^n}}}}}})$
- $\mathbf{R}$: recursive languages

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## Beyond NEXP—cont'd

- **NEXP**/**EXP**-complete problems are provably not in **P**.
- The exponential hierarchy is a provably true hierarchy.
- There are natural problems even in the higher classes:
  Regular expression equivalence (with squaring) is **EXPSPACE**-complete.
  Regular expression equivalence (with squaring and complement) is not even in **ELEMENTARY**.

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## Computational complexity theory

- **L** $\subseteq$ **NL** $\subseteq$ **P** $\subseteq$ **NP** $\subseteq$ **PSPACE** $\subseteq$ **EXP** $\subseteq$ **NEXP** $\subseteq$ **EXPSPACE** $\subseteq$ 2-**EXP** $\subseteq$ **ELEMENTARY** $\subseteq$ **R**
- Reductions:
  $A$ reduces to $B \longleftrightarrow$ "$B$ is at least as hard as $A$"
  (i) Positive result: For solving $A$ we can use an algorithm for $B$.
  (ii) Negative result: If $A$ is hard then so is $B$.
- Complete problems:
  Hardest among those in the same class
  Least likely to belong to a lower class.

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- When finding a hard problem:
  - Analyze the borderline of hardness
  - Study special cases
  - Efficient search methods and good heuristics
  - Approximations
  - Randomized algorithms
  - Local search methods

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