

## Instructions

- Remember to justify your answers, and be precise. A good way of replying is to explain your solution as you would to a B.Sc. student studying CS at TKK.
- The problems are to be solved *individually* – *no plagiarism* is tolerated. If you discuss the exercises with other students, please mention their names.

You can hand in your answers by

- handing them to the lecturer at lectures, or
- slipping them into the post box in-between rooms B336 and B337, or
- sending them by e-mail to the assistant (mjj [AT] tcs.tkk.fi). In this case the only acceptable file formats are postscript, pdf, and ASCII text. Also, please start the Subject-header with “T-79.5103”.

Write all of the following information on the first sheet:

- Course code, name, and year  
"T-79.5103 Computational Complexity Theory 2007"
- Your name and student ID
- Home assignment round (e.g., HA 1)

If you return answers on paper:

- Use paper size A4 only
- If you return multiple sheets, bind them together with e.g. staples
- Please take copies of your sheets in case you want to save them.

If you return answers by email:

- Use the filename lastname-homeassignmentround.fileformat.  
For example: jarvisalo-1.ps
- Only file formats ps, pdf, and txt are allowed

## Reviewing and Revising Your Answers

You can get feedback on your solution with the assistant on 10.12. at 10:00 in room T-B354 (the assistant’s office). If you get less than 1.5 points for some exercise, you then have the possibility to revise your answer. The **deadline for submitting revised answers is 21.12.2007**. Revised answer will be graded using the scale 0 – 1.5.

**Deadline for these exercises is 5.12.2007**

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## Exercises (Chapters 11–19)

### 3.1 (0-2 points)

- (i) Show that **RP** is closed under intersection.
- (ii) Show that **BPP** is closed under union.

### 3.2 (0-2 points)

Prove the following claims.

- (a) Unless  $\mathbf{P} = \mathbf{NP}$ , the approximation threshold of MINIMUM COLORING cannot be less than  $\frac{1}{4}$ .
- (b) CREW PRAM can compute the OR of  $n$  bits in  $O(\log n)$  time with  $O(n)$  processors.

### 3.3 (0-2 points)

Prove the following claims.

- (a) CRITICAL 3-COLORABILITY  $\in \mathbf{DP}$
- (b) UNIQUE SAT  $\in \mathbf{DP}$

### 3.4 (0-2 points)

*We know that converting Boolean expressions in DNF to CNF can be exponential in the worst case (because the output may be exponentially long in the size of the input). But, suppose that the output is small. In particular:*

Show that the following problem is in  $\Sigma_2\mathbf{P}$ : Given an integer  $B$  and a Boolean expression  $D$  in disjunctive normal form (DNF), is there an expression  $C$  in conjunctive normal form (CNF) which is equivalent to  $D$  and has at most  $B$  clauses?

### 3.5 (0-2 points)

Show that the following problem is **PSPACE**-complete: Given a deterministic Turing machine  $M$  and an integer  $k$ , is there an input  $x$  of length  $k$  such that  $M$  accepts  $x$  without ever leaving the  $k + 1$  first symbols of its string?

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