1. Consider the query $P(Rain \mid Sprinkler = true, WetGrass = true)$ for the network given below and how MCMC can answer it.

(a) How many states does the Markov chain have?
(b) Calculate the transition matrix $Q$ containing $q(x \rightarrow x')$ for all states $x$ and $x'$.
(c) What does the square of the transition matrix $Q^2$ represent?
(d) What about $Q^n$ as $n \rightarrow \infty$.

(R&N, Exercise 14.11.abcd)

2. A fire station has one fire truck. Upon an emergency call, the truck goes out to fight fire and then returns to the station.

(a) Design a hidden Markov model (HMM) with two states $s$ (the truck is at the fire station) and $\neg s$ to describe the behaviour of this system. Choose transition probabilities to reflect the following properties of the domain.
   — On the average, there is an alert once in twelve hours.
   — The expected duration for one fire mission is 3 hours.
   Use one hour time slices in your model.
(b) Write down the corresponding transition matrix $Q$ for the HMM as well as the transition model

$$P(FS_{t+1} \mid FS_t)$$

using a Boolean random variable $FS$.

(c) Use the model to determine how many hours a day the truck spends at the fire station in the long run?
(d) Given a parameterised prior distribution $P(FS_0) = (r, 1-r)$, derive an exact expression for the distribution $P(FS_t)$ as a function of $t$ using the transition model and prediction.
(e) Does $P(FS_t)$ converge as $t$ approaches to infinity?