T-79.5102 **Autumn 2007**
Special Course in Computational Logic
Tutorial 11

1. Consider a normal logic program P consisting of the following rules.

$$a \leftarrow b,\, d. \quad a \leftarrow {\sim}d. \quad b \leftarrow c,\, {\sim}e. \quad c \leftarrow a. \quad d \leftarrow {\sim}e. \quad e \leftarrow {\sim}d$$

Determine the following sets of interpretations for P:

(a) $\mathrm{CM}(P)$,

(b) $\mathrm{SuppM}(P)$, and

(c) $\mathrm{SM}(P)$.

2. Form $\mathrm{Comp}(P)$ and $\mathrm{LoopF}(P)$ for the program addressed in the previous assignment. Then calculate

(a) $\mathrm{CM}(\mathrm{Comp}(P))$ and

(b) $\mathrm{CM}(\mathrm{Comp}(P) \cup \mathrm{LoopF}(P))$.

3. Analyze the normal program P_n given in Figure 1 and prove that P_n is tight on a supported model $M =$

$$\begin{array}{l}\{\mathsf{Node}(x) \mid 0 \leq x \leq n\} \cup \{\mathsf{Edge}(x, x+1) \mid 0 \leq x < n\} \cup \\ \{\mathsf{Edge}(n, 0), \mathsf{Out}(n, 0)\} \cup \{\mathsf{In}(x, x+1) \mid 0 \leq x < n\} \cup \\ \{\mathsf{Reach}(x, y) \mid 0 \leq x < y \leq n\}.\end{array}$$

Provide a concrete numbering $\lambda : M \rightarrow \mathbb{N}$ in your proof.

4. Continue the analysis of P_n from Figure 1:

(a) Identify loops in $\mathrm{DG}^{+}(\mathrm{Gnd}(P_n))$.

(b) What can be stated about the respective disjunctive loop formulas $\mathrm{LoopF}(L, \mathrm{Gnd}(P_n))$?

$\mathsf{Edge}(0,1). \quad \ldots \quad \mathsf{Edge}(n-1,n). \quad \mathsf{Edge}(n,0).$
$\mathsf{In}(x,y) \leftarrow {\sim}\mathsf{Out}(x,y),\, \mathsf{Edge}(x,y). \quad \mathsf{Out}(x,y) \leftarrow {\sim}\mathsf{In}(x,y),\, \mathsf{Edge}(x,y).$
$\mathsf{F} \leftarrow \mathsf{In}(0,1),\, \ldots,\, \mathsf{In}(n-1,n),\, \mathsf{In}(n,0),\, {\sim}\mathsf{F}.$
$\mathsf{F} \leftarrow \mathsf{Out}(x,y),\, \mathsf{Out}(z,v),\, {\sim}\mathsf{F},\, \mathsf{Edge}(x,y),\, \mathsf{Edge}(z,v),\, x \neq z.$
$\mathsf{Reach}(x,y) \leftarrow \mathsf{In}(x,y),\, \mathsf{Edge}(x,y). \quad \mathsf{Node}(x) \leftarrow \mathsf{Edge}(x,y).$
$\mathsf{Reach}(x,y) \leftarrow \mathsf{Reach}(x,z), \mathsf{In}(z,y),\, \mathsf{Node}(x),\, \mathsf{Edge}(z,y),\, y \neq z.$

Figure 1: Example of a tight program