T-79.5102 Autumn 2007

Special Course in Computational Logic Tutorial 11

1. Consider a normal logic program P consisting of the following rules.

$$a \leftarrow b, d.$$
 $a \leftarrow \sim d.$ $b \leftarrow c, \sim e.$ $c \leftarrow a.$ $d \leftarrow \sim e.$ $e \leftarrow \sim d$

Determine the following sets of interpretations for P:

- (a) CM(P),
- (b) SuppM(P), and
- (c) SM(P).
- **2.** Form Comp(P) and LoopF(P) for the program addressed in the previous assignment. Then calculate
 - (a) CM(Comp(P)) and
 - (b) $CM(Comp(P) \cup LoopF(P))$.
- **3.** Analyze the normal program P_n given in Figure 1 and prove that P_n is tight on a supported model M =

$$\begin{cases} \mathsf{Node}(x) \mid 0 \le x \le n \} \cup \{ \mathsf{Edge}(x, x+1) \mid 0 \le x < n \} \cup \\ \{ \mathsf{Edge}(n, 0), \mathsf{Out}(n, 0) \} \cup \{ \mathsf{In}(x, x+1) \mid 0 \le x < n \} \cup \\ \{ \mathsf{Reach}(x, y) \mid 0 \le x < y \le n \}. \end{cases}$$

Provide a concrete numbering $\lambda: M \to \mathbb{N}$ in your proof.

- **4.** Continue the analysis of P_n from Figure 1:
 - (a) Identify loops in $DG^+(Gnd(P_n))$.
 - (b) What can be stated about the respective disjunctive loop formulas $\text{LoopF}(L, \text{Gnd}(P_n))$?

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\begin{split} &\mathsf{Edge}(0,1). \quad \dots \; \mathsf{Edge}(n-1,n). \quad \mathsf{Edge}(n,0). \\ &\mathsf{In}(x,y) \leftarrow \sim \mathsf{Out}(x,y), \; \mathsf{Edge}(x,y). \quad \mathsf{Out}(x,y) \leftarrow \sim \mathsf{In}(x,y), \; \mathsf{Edge}(x,y). \\ &\mathsf{F} \leftarrow \mathsf{In}(0,1), \dots, \mathsf{In}(n-1,n), \; \mathsf{In}(n,0), \; \sim \mathsf{F}. \\ &\mathsf{F} \leftarrow \mathsf{Out}(x,y), \; \mathsf{Out}(z,v), \; \sim \mathsf{F}, \; \mathsf{Edge}(x,y), \; \mathsf{Edge}(z,v), \; x \neq z. \\ &\mathsf{Reach}(x,y) \leftarrow \mathsf{In}(x,y), \; \mathsf{Edge}(x,y). \quad \mathsf{Node}(x) \leftarrow \mathsf{Edge}(x,y). \\ &\mathsf{Reach}(x,y) \leftarrow \mathsf{Reach}(x,z), \; \mathsf{In}(z,y), \; \mathsf{Node}(x), \; \mathsf{Edge}(z,y), \; y \neq z. \end{split}
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Figure 1: Example of a tight program