T-79.5102 **Autumn 2007**
Special Course in Computational Logic
Tutorial 10

1. The classical semantics of a propositional theory T is determined by $\mathrm{CM}(T) = \{M \subseteq \mathrm{Hb}(T) \mid M \models T\}$. The modularity of $\mathrm{CM}(\cdot)$ can be formalized as an equation

$$\mathrm{CM}(T_1 \cup T_2) = \mathrm{CM}(T_1) \bowtie \mathrm{CM}(T_2) \tag{1}$$

where $\bowtie$ combines any pair of interpretations $M_1 \subseteq \mathrm{Hb}(T_1)$ and $M_2 \subseteq \mathrm{Hb}(T_2)$ which are *compatible*, i.e., $M_1 \cap \mathrm{Hb}(T_2) = M_2 \cap \mathrm{Hb}(T_1)$, into $M_1 \cup M_2 \subseteq \mathrm{Hb}(T_1 \cup T_2)$.

(a) Prove (1) for any propositional theories T_1 and T_2.

(b) Generalize (1) for any number of propositional theories $T_1, \ldots, T_n$.

(c) Apply the generalized form of (1) to propositional theories

$$T_1 = \{r_1 \rightarrow r_2\},\ T_2 = \{r_2 \rightarrow r_3\},\ \ldots,\ T_{n-1} = \{r_{n-1} \rightarrow r_n\},$$

and $T_n = \{r_n \rightarrow r_1\}$, i.e., to calculate $\mathrm{CM}(T_1 \cup \ldots \cup T_n)$.

2. Consider the following `smodels` program modules:

$\{a\}$
$a \leftarrow b.$
$a \leftarrow c, \sim b.$
$b \leftarrow c, \sim a.$
$\{c\}$

$\{b\}$
$b \leftarrow d, \sim d.$
$d \leftarrow c, \sim b.$
$d \leftarrow c, \sim d.$
$\{c\}$

$\{c\}$
$c \leftarrow a.$
$c \leftarrow \sim a, \sim b.$
$c \leftarrow c, \sim a.$
$\{a, b\}$

$\{c, d\}$
$c \leftarrow \sim d.$
$d \leftarrow e, \sim c.$
$d \leftarrow c, \sim e.$
$\{e\}$

(a) For which pairs of modules are compositions and joins defined?

(b) Apply the module theorem, i.e., the equation

$$\mathrm{SM}(\mathbb{P} \sqcup \mathbb{Q}) = \mathrm{SM}(\mathbb{P}) \bowtie \mathrm{SM}(\mathbb{Q}), \tag{2}$$

to some pair of modules $\mathbb{P}$ and $\mathbb{Q}$ for which $\mathbb{P} \sqcup \mathbb{Q}$ is defined.

3. Prove the following algebraic properties of $\oplus$ under the assumption that designated leftmost compositions are defined:

(a) $\mathbb{P} \oplus \varnothing = \varnothing \oplus \mathbb{P} = \mathbb{P}$ where $\varnothing$ denotes an empty module $\langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$.

(b) $\mathbb{P} \oplus \mathbb{Q} = \mathbb{Q} \oplus \mathbb{P}$.

(c) $(\mathbb{P} \oplus \mathbb{Q}) \oplus \mathbb{R} = \mathbb{P} \oplus (\mathbb{Q} \oplus \mathbb{R})$.

What if $\oplus$ is replaced by $\sqcup$ in the equations above?

4. Recall that $P = \{a \leftarrow b.\ \ a \leftarrow \sim b.\ \}$ and $Q = \{a.\ \}$ are not strongly equivalent. What about the modular equivalence of the respective program modules $\mathbb{P} = \langle P, \{b\}, \{a\}, \emptyset \rangle$ and $\mathbb{Q} = \langle Q, \{b\}, \{a\}, \emptyset \rangle$?