T-79.5102 **Autumn 2007**
Special Course in Computational Logic
Tutorial 7

1. Justify the following strategy for establishing the NP-completeness of a language L in terms of reductions.

1. For membership, L is reduced to another NP-complete language L_1.
2. For hardness, an NP-complete language L_2 is reduced to L.

What can be said about the complexity of L if any of the objectives fails?

2. The classification of $\mathsf{CAUTIOUS}$ involved two reductions:

$$\begin{aligned} \langle P, a\rangle \notin \mathsf{CAUTIOUS} &\iff R_1(P,a) = P \cup \{f \leftarrow a, \sim f.\} \in \mathsf{STABLE} \\ P \in \mathsf{STABLE} &\iff R_2(P) = \langle P \cup \{f \leftarrow f.\}, f\rangle \notin \mathsf{CAUTIOUS} \end{aligned}$$

In both cases, the atom f introduced by the reduction is new, i.e., $f \notin \mathrm{Hb}(P)$. Prove the correctness of the reductions R_1 and R_2.

3. Analyze the computational time complexity of decision problems STABLE, BRAVE, and $\mathsf{CAUTIOUS}$ in the case of positive programs.

4. Given a normal program P, the transfinite iteration sequence of Γ_P^2 is defined for all ordinals α as follows:

(i) $\Gamma_P^2 \uparrow 0 = \emptyset$.
(ii) $\Gamma_P^2 \uparrow \alpha + 1 = \Gamma_P^2(\Gamma_P^2 \uparrow \alpha)$ for a successor ordinal $\alpha + 1$.
(iii) $\Gamma_P^2 \uparrow \alpha = \bigcup_{\beta<\alpha} \Gamma_P^2 \uparrow \beta$ for a limit ordinal α.

Prove in detail that $\Gamma_P^2 \uparrow \alpha \subseteq M \subseteq \Gamma_P(\Gamma_P^2 \uparrow \alpha)$ for any α and any stable model $M \in \mathrm{SM}(P)$.

5. Determine $\mathrm{SM}(P)$ and $\mathrm{WFM}(P)$ for normal programs P consisting of

(a) $a \leftarrow \sim b.\quad b \leftarrow \sim c.\quad c \leftarrow \sim a.$
(b) $a \leftarrow \sim b.\quad b \leftarrow \sim c.\quad b \leftarrow \sim d.\quad c \leftarrow \sim a.\quad e \leftarrow a.\quad e \leftarrow \sim c.\quad f \leftarrow \sim e.$
(c) $a \leftarrow \sim b.\quad b \leftarrow \sim a.\quad c \leftarrow a, b.\quad d \leftarrow a.\quad d \leftarrow b.\quad e \leftarrow d, \sim c.$