T-79.5102 **Autumn 2007**
Special Course in Computational Logic
Tutorial 1

1. Consider the following propositional theory

$$T_n = \{r_1 \rightarrow r_2,\ r_2 \rightarrow r_3,\ \ldots,\ r_{n-1} \rightarrow r_n,\ r_n \rightarrow r_1\}$$

based on $\mathcal{P}_n = \{r_1, \ldots, r_n\}$ where $n > 0$. The intuitive reading of r_i is that the i^{th} node is *reachable* in a finite ring of n nodes.

(a) Determine the models $M \subseteq \mathcal{P}_n$ of T_n.

(b) Show/argue that $T \models r_i \rightarrow r_j$ for any $0 < i, j \leq n$.

(c) Compare the models of T_n with the models of $T_n \cup \{r_1\}$? Does T_n properly formalize reachability among the nodes in the ring?

2. Consider a set of nodes $U_n = \{1, \ldots, n\}$ and a relation $E \subseteq (U_n)^2$ representing the edge relation of a graph $G = \langle U_n, E\rangle$ with n vertices. Which properties of G are captured by the following first-order sentences?

(a) $\phi_1 = \forall x \exists y E(x, y) \wedge \forall x \forall y \forall z (E(x, y) \wedge E(x, z) \rightarrow y = z)$

(b) $\phi_2 = \forall x \exists y E(y, x) \wedge \forall x \forall y \forall z (E(y, x) \wedge E(z, x) \rightarrow y = z)$

(c) $\phi_3 = \phi_1 \wedge \phi_2$

(d) $\phi_4 = \forall x E(x, x)$

(e) $\phi_5 = \forall x \forall y (E(x, y) \rightarrow E(y, x))$

(f) $\phi_6 = \forall x \forall y \forall z (E(x, z) \wedge E(z, y) \rightarrow E(x, y))$

(g) $\phi_7 = \phi_4 \wedge \phi_5 \wedge \phi_6$

3. Describe the models of ϕ_3 and ϕ_7 (see above) when $n = 3$.

4. Consider the following first-order theory

$$T = \{N(0, s(0)),\ \forall x \forall y (N(x, y) \rightarrow N(s(x), s(y)))\}$$

which captures for each natural number its immediate successor.

(a) Determine the Hebrand base $\mathrm{Hb}(T)$.

(b) Determine a Herbrand model $M \subseteq \mathrm{Hb}(T)$ for T.

(c) Is M minimal with respect to $\subseteq$, i.e., is there another Herbrand interpretation $M' \subset M$ of T such that $M' \models T$?