T-79.5102 **Autumn 2006**
Special Course in Computational Logic
Tutorial 11
Solutions

1. The CURRENT-BEST-LEARNING algorithm produces hypotheses listed in the following table (not necessarily unique). In the second column, FP and FN denote *false positive* and *false negative* examples, respectively.

(a) Starting from the hypothesis $\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Hun}(x))$:

Ex		Hypothesis:
x_1		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Hun}(x))$
x_2	FP	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Hun}(x) \wedge \mathit{Est}(x, \mathit{0\text{-}10}))$
x_3	FN	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, \mathit{0\text{-}10}))$
x_4	FN	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, \mathit{0\text{-}10}) \vee \mathit{Est}(x, \mathit{10\text{-}30}))$
x_5		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, \mathit{0\text{-}10}) \vee \mathit{Est}(x, \mathit{10\text{-}30}))$
x_6		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, \mathit{0\text{-}10}) \vee \mathit{Est}(x, \mathit{10\text{-}30}))$
x_7	FP	$\forall x(\mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, \mathit{0\text{-}10}) \wedge \mathit{Pat}(x, \mathit{some}))$ $\vee \mathit{Est}(x, \mathit{10\text{-}30}))$
x_8		$\forall x(\mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, \mathit{0\text{-}10}) \wedge \mathit{Pat}(x, \mathit{some}))$ $\vee \mathit{Est}(x, \mathit{10\text{-}30}))$
x_9		$\forall x(\mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, \mathit{0\text{-}10}) \wedge \mathit{Pat}(x, \mathit{some}))$ $\vee \mathit{Est}(x, \mathit{10\text{-}30}))$
x_{10}	FP	$\forall x(\mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, \mathit{0\text{-}10}) \wedge \mathit{Pat}(x, \mathit{some})) \vee$ $(\mathit{Est}(x, \mathit{10\text{-}30}) \wedge \neg \mathit{Price}(x, \$\$\$)))$
x_{11}		$\forall x(\mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, \mathit{0\text{-}10}) \wedge \mathit{Pat}(x, \mathit{some})) \vee$ $(\mathit{Est}(x, \mathit{10\text{-}30}) \wedge \neg \mathit{Price}(x, \$\$\$)))$
x_{12}	FN	$\forall x(\mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, \mathit{0\text{-}10}) \wedge \mathit{Pat}(x, \mathit{some})) \vee$ $(\mathit{Est}(x, \mathit{10\text{-}30}) \wedge \neg \mathit{Price}(x, \$\$\$)) \vee$ $(\mathit{Est}(x, \mathit{30\text{-}60}) \wedge \mathit{Type}(x, \mathit{burger})))$

(b) For the hypothesis $\forall x(\mathit{WillWait}(x) \leftrightarrow \neg \mathit{Est}(x, \mathit{30\text{-}60}))$:

Ex		Hypothesis:
x_1	FN	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, \mathit{30\text{-}60}) \vee \mathit{Pat}(x, \mathit{some}))$
x_2	FP	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}))$
x_3		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}))$
x_4	FN	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Hun}(x))$
x_5		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Hun}(x))$
x_6		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Hun}(x))$
x_7		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Hun}(x))$
x_8		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Hun}(x))$
x_9		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Hun}(x))$
x_{10}	FP	$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee$ $(\mathit{Hun}(x) \wedge \neg \mathit{Price}(x, \$\$\$)))$
x_{11}		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee$ $(\mathit{Hun}(x) \wedge \neg \mathit{Price}(x, \$\$\$)))$
x_{12}		$\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee$ $(\mathit{Hun}(x) \wedge \neg \mathit{Price}(x, \$\$\$)))$

Note: the choices made above are not unique (other solutions exist)!

2. In the candy domain, hypotheses are based on the following five mixings of the two flavours:

h_1: 100% cherry

h_2: 75% cherry and 25% lime

h_3: 50% cherry and 50% lime

h_4: 25% cherry and 75% lime

h_5: 100% lime

After unwrapping four pieces of the surprise candy we note that three pieces are cherry flavoured.

(a) To find out the most likely (ML) hypothesis let us denote the observed data, i.e., three cherry candies and one lime candy, by $\mathbf{d}$. The probability of $\mathbf{d}$ (given each hypothesis h_i in turn) varies as follows:

$$\begin{aligned}
&P(\mathbf{d} \mid h_1) = 0 \\
&P(\mathbf{d} \mid h_2) = 4 \cdot (\tfrac{3}{4})^3 \cdot \tfrac{1}{4} = \tfrac{27}{64} \approx 0.42 \\
&P(\mathbf{d} \mid h_3) = 4 \cdot (\tfrac{1}{2})^3 \cdot \tfrac{1}{2} = 0.25 \\
&P(\mathbf{d} \mid h_4) = 4 \cdot (\tfrac{1}{4})^3 \cdot \tfrac{3}{4} = \tfrac{3}{64} \approx 0.047 \\
&P(\mathbf{d} \mid h_5) = 0
\end{aligned}$$

Thus h_2 is the most likely hypothesis.

(b) Let us then suppose that the prior distribution of the bags is

$$\langle 0.1, 0.1, 0.1, 0.6, 0.1 \rangle.$$

In order to determine the maximum a posteriori (MAP) hypothesis we calculate:

$$\begin{aligned}
&P(h_1) = 0.1 \\
&P(h_2) = 0.1 \\
&P(h_3) = 0.1 \\
&P(h_4) = 0.6 \\
&P(h_5) = 0.1 \\
&P(h_1 \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_1) P(h_1) = 0 \\
&P(h_2 \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_2) P(h_2) \approx 0.042\alpha \\
&P(h_3 \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_3) P(h_3) \approx 0.025\alpha \\
&P(h_4 \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_4) P(h_4) \approx 0.028\alpha \\
&P(h_5 \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_5) P(h_5) = 0
\end{aligned}$$

$$\begin{aligned}
&P(h_1 \mid \mathbf{d}) = 0 \\
&P(h_2 \mid \mathbf{d}) = \tfrac{0.042}{0.042+0.025+0.028} \approx 0.44 \\
&P(h_3 \mid \mathbf{d}) = \tfrac{0.025}{0.042+0.025+0.028} \approx 0.26 \\
&P(h_4 \mid \mathbf{d}) = \tfrac{0.028}{0.042+0.025+0.028} \approx 0.29 \\
&P(h_5 \mid \mathbf{d}) = 0
\end{aligned}$$

We conclude that the maximum a posteriori hypothesis is h_2, i.e., the prior distribution does not affect the dominating hypothesis.

(c) Next we estimate the probability that the fifth piece of candy is lime:

$$\begin{aligned}
P(\mathit{lime} \mid \mathbf{d}) = \; & P(\mathit{lime} \mid h_2) P(h_2 \mid \mathbf{d}) + P(\mathit{lime} \mid h_3) P(h_3 \mid \mathbf{d}) + \\
& P(\mathit{lime} \mid h_4) P(h_4 \mid \mathbf{d}).
\end{aligned}$$

Using the prior distribution of bags given in item (b) we may calculate the probability of getting a lime for the fifth draw:

$$P(\text{lime} \mid \mathbf{d}) = 0.25 \cdot 0.44 + 0.5 \cdot 0.26 + 0.75 \cdot 0.29 \approx 0.46.$$