T-79.5102 **Autumn 2006**
Special Course in Computational Logic
Tutorial 6
Solutions

1. (a) The costs of the three routes are as follows:

Route	Time (min)	Fares (mk)
I	57	39
II	33	26
III	55	20

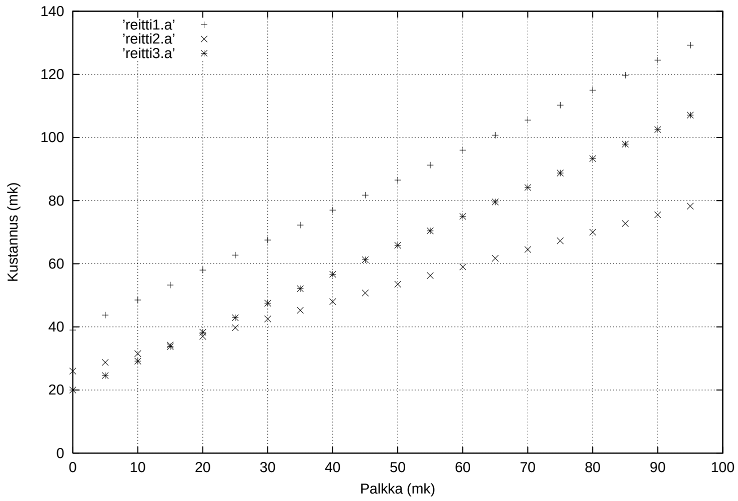
If the engineer's hourly salary is $a = 40$ mk, the values of the cost function $U(t, m) = m + at$ are as follows:

$$\begin{aligned}\text{I: } U(57, 39) &= 39 + \frac{57}{60} \cdot 40 = 77.00 \text{ (mk)}\\ \text{II: } U(33, 26) &= 26 + \frac{33}{60} \cdot 40 = 48.00 \text{ (mk)}\\ \text{III: } U(55, 20) &= 20 + \frac{55}{60} \cdot 40 = 56.70 \text{ (mk)}\end{aligned}$$

These values indicate that the second route is the best alternative. The point where III becomes better than II can be found by solving the following inequality:

$$\tfrac{33}{60}x + 26 \geq \tfrac{55}{60}x + 20 \iff x \leq 16.36 \quad \text{(mk/h)}.$$

Thus the engineer should earn less than 16.36 mk/h to make route III a cheaper one. When a varies in the range 0–100, the respective costs for the three routes have been plotted in the figure given below.



As regards costs, we note that I dominates (yields higher costs in any event) the two other routes so that it can be safely ignored by the engineer.

(b) Let us then introduce a revised cost function

$$U(t_1, t_2, m) = a_1 t_1 + a_2 t_2 + m$$

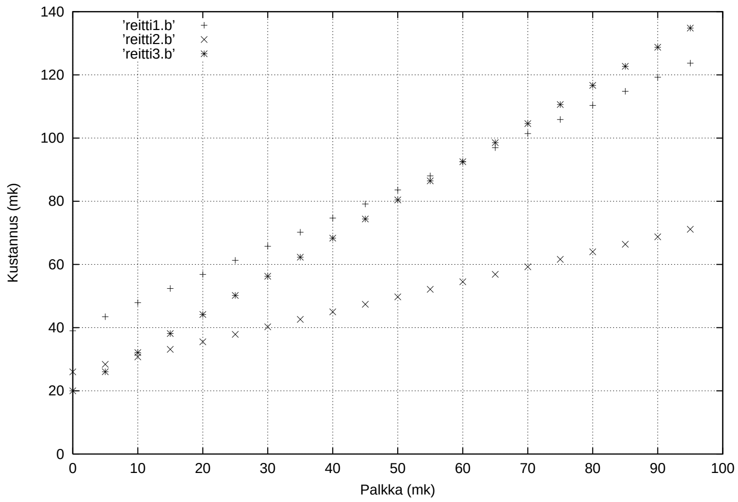
with parameters $a_1 = 1.5a$ and $a_2 = 0.5a$. The following times and ticket fares are associated with the routes under consideration:

Route	Time t_1 (min)	Time t_2 (min)	Fares (mk)
I	25	32	39
II	12	21	26
III	45	10	20

Thus, the overall costs of the routes are:

$$\text{I: } U(25,32,39) = 39 + \frac{25}{60}\cdot 60 + \frac{32}{60}\cdot 20 = 74.70 \text{ (mk)}$$
$$\text{II: } U(12,21,26) = 26 + \frac{12}{60}\cdot 60 + \frac{21}{60}\cdot 20 = 45.00 \text{ (mk)}$$
$$\text{III: } U(45,10,20) = 20 + \frac{45}{60}\cdot 60 + \frac{10}{60}\cdot 20 = 68.33 \text{ (mk)}$$

Again, the second route turned out to be better than the others. The following figure shows how costs change as the function of the engineer's hourly salary:



Therefore, none of the options dominates within this interval.

(c) If the outcomes of choices made by the engineer are not deterministic, we use the expected utility $E[U(X)]$ as the basis for decisions. The probability distributions for the three options are:

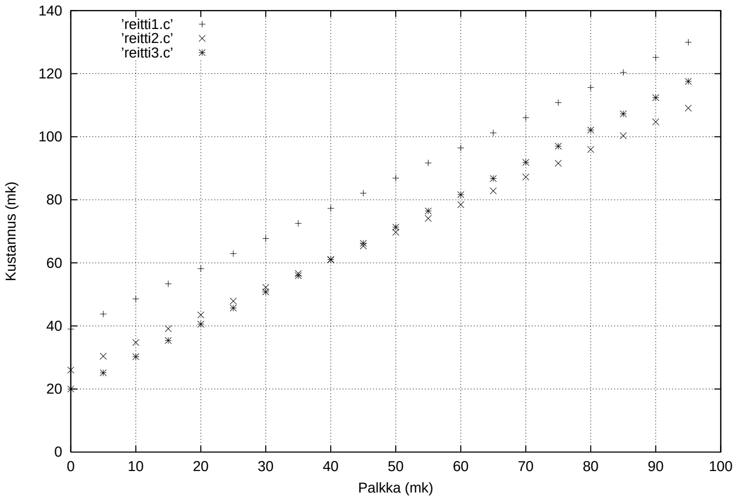
Route	Time t (min)	$P(t)$
I	57	0.75
	58	0.20
	62	0.05
II	33	0.30
	34	0.20
	43	0.20
	48	0.30

Route	Time t (min)	$P(t)$
III	55	0.16
	56	0.19
	57	0.03
	60	0.17
	61	0.04
	65	0.17
	66	0.03
	70	0.17
	71	0.03
	75	0.01

These lead to the following expected values and costs:

Route	$E(t)$ (min)	$U(t,m)$ (mk)
I	57.45	77.3
II	39.7	52.47
III	61.6	61.06

Thus II is again the leading option for the engineer.


'reitti1.c'
'reitti2.c'
'reitti3.c'
Kustannus (mk)
Palkka (mk)
0
20
40
60
80
100
120
140
0
10
20
30
40
50
60
70
80
90
100