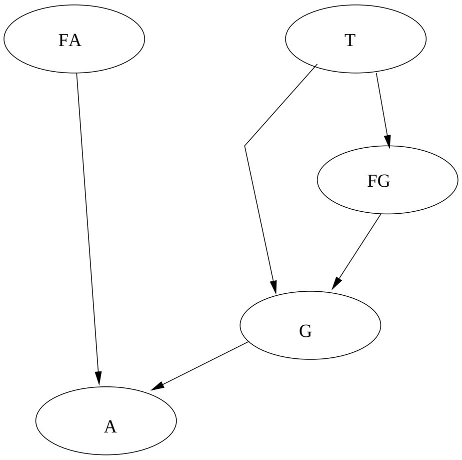
T-79.5102 **Autumn 2006**
Special Course in Computational Logic
Tutorial 3
Solutions

1. (a) We start by abstracting temperatures so that there are only two possible values high and normal (i.e., not high). These values are captured by the Boolean variable T below: if $T = \mathit{true}$, then the temperature is too high, and if $T = \mathit{false}$, the temperature is normal.

We use the following nodes (random variables) in the network:

- F_A — The alarm is faulty.
- T — The temperature of the core is too high.
- F_G — The gauge is faulty.
- G — The gauge shows a high temperature.
- A — The alarm goes off.

Each variable is a Boolean one, i.e., takes T or F as its value. The dependencies described in the exercise text lead us to construct the following Bayesian network as a model of the domain:



(b) The network is not a polytree, since there are two different paths from variable T to variable G.

(c) See (a) for the abstraction of temperatures, i.e., the values of variable T. The CPT associated with G is the following:

T	F_G	$P(G)$	$P(\neg G)$
T	T	y	$1-y$
T	F	x	$1-x$
F	T	$1-y$	y
F	F	$1-x$	x

(d) The CPT associated with A is given below:

G	F_A	$P(A)$	$P(\neg A)$
T	T	0	1
T	F	1	0
F	T	0	1
F	F	0	1

Thus we may conclude that there is a logical relationship among the three variables involved: $A \leftrightarrow G \wedge \neg F_A$.

(e) The distribution $\mathbf{P}(T \mid \neg f_A, \neg f_G, a)$ can be determined for instance as follows:

$$
\begin{array}{lll}
 & \mathbf{P}(T \mid a, \neg f_A, \neg f_G) & \\
= & \mathbf{P}(T \mid a, \neg f_A, g, \neg f_G) & \{a \leftrightarrow g \wedge \neg f_A, \neg f_A, a\} \models g \\
= & \mathbf{P}(T \mid g, \neg f_G) & \text{Cond. Ind. } mb(T) = \{F_G, G\} \\
= & \alpha \mathbf{P}(g, \neg f_G \mid T)\mathbf{P}(T) & \text{Bayes \& Normalization} \\
= & \alpha \mathbf{P}(g, \neg f_G, T) & \text{Cond. prob.} \\
= & \alpha \mathbf{P}(g \mid \neg f_G, T)\mathbf{P}(\neg f_G \mid T)\mathbf{P}(T). & \text{Network semantics}
\end{array}
$$

From this we obtain an expression for $P(t \mid a, \neg f_A, \neg f_G)$ by normalization, i.e., $1/\alpha$ is the sum of the two probability expressions:

$$\frac{P(g \mid \neg f_G, t)P(\neg f_G \mid t)P(t)}{P(g \mid \neg f_G, t)P(\neg f_G \mid t)P(t) + P(g \mid \neg f_G, \neg t)P(\neg f_G \mid \neg t)P(\neg t)}$$

which could also be rewritten as

$$\frac{1}{1 + \frac{P(g \mid \neg f_G, \neg t)P(\neg f_G \mid \neg t)P(\neg t)}{P(g \mid \neg f_G, t)P(\neg f_G \mid t)P(t)}}.$$

If we substitute the known known probability values from the CPTs given above and extend the resulting fraction by x, we obtain

$$\frac{x}{x + (1-x)\frac{P(\neg f_G \mid \neg t)P(\neg t)}{P(\neg f_G \mid t)P(t)}}.$$