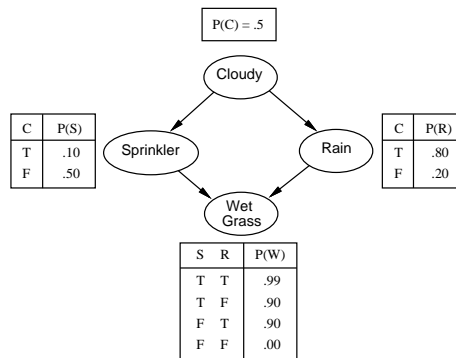


1. Consider the query $\mathbf{P}(\text{Rain} \mid \text{Sprinkler} = \text{true}, \text{WetGrass} = \text{true})$ for the network given below and how MCMC can answer it.
 - (a) How many states does the Markov chain have?
 - (b) Calculate the transition matrix Q containing $q(\mathbf{x} \rightarrow \mathbf{x}')$ for all states \mathbf{x} and \mathbf{x}' .
 - (c) What does the square of the transition matrix Q^2 represent?
 - (d) What about Q^n as $n \rightarrow \infty$.



(R&N, Exercise 14.11.abcd)

2. A fire station has one fire truck. Upon an emergency call, the truck goes out to fight fire and then returns to the station.
 - (a) Design a hidden Markov model (HMM) with two states fs (the truck is at the fire station) and $\neg fs$ to describe the behaviour of this system. Choose transition probabilities to reflect the following properties of the domain.
 - On the average, there is an alert once in twelve hours.
 - The expected duration for one fire mission is 3 hours.
 Use one hour time slices in your model.
 - (b) Write down the corresponding transition matrix Q for the HMM as well as the transition model

$$\mathbf{P}(FS_{t+1} \mid FS_t)$$

using a Boolean random variable FS .

- (c) Use the model to determine how many hours a day the truck spends at the fire station in the long run?
- (d) Given a parametrised prior distribution $\mathbf{P}(FS_0) = \langle r, 1 - r \rangle$, derive an exact expression for the distribution $\mathbf{P}(FS_t)$ as a function of t using the transition model and prediction.
- (e) Does $\mathbf{P}(FS_t)$ converge as t approaches to infinity?