T-79.5101 **Spring 2008**
Advanced Course in Computational Logic
Exercise Session 8

1. Translate the formula $\Diamond\Box\Diamond P \rightarrow \Diamond P$ into predicate logic. Using the translated formula and tableaux for predicate logic, show that the original formula is **KB**-valid.

2. Let $\mathcal{M} = \langle S, R_1, R_2, R_3, v\rangle$, where

$S = \{s_1, s_2, s_3, s_4\}$,
$R_1 = \{\langle s_1, s_1\rangle, \langle s_2, s_2\rangle, \langle s_3, s_3\rangle, \langle s_4, s_4\rangle, \langle s_1, s_2\rangle, \langle s_2, s_1\rangle\}$,
$R_2 = \{\langle s_1, s_1\rangle, \langle s_2, s_2\rangle, \langle s_3, s_3\rangle, \langle s_4, s_4\rangle, \langle s_3, s_2\rangle, \langle s_2, s_3\rangle\}$,
$R_3 = \{\langle s_1, s_1\rangle, \langle s_2, s_2\rangle, \langle s_3, s_3\rangle, \langle s_4, s_4\rangle, \langle s_3, s_4\rangle, \langle s_4, s_3\rangle\}$ ja
$v(s_1, P) = v(s_2, P) = v(s_3, P) = \text{true}$, $v(s_4, P) = \text{false}$.

Which of the following claims hold in the logic $\mathbf{S5_3}$?
a) $\mathcal{M}, s_1 \Vdash EP$
b) $\mathcal{M}, s_1 \Vdash EEP$
c) $\mathcal{M}, s_1 \Vdash CP$.

3. Show that the following holds for any formula ϕ: If ϕ is not **S5**-valid, then there is a a counter-model for ϕ (a model in which ϕ is false) having at most as many possible worlds as there are subformulas of ϕ.

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Exercise Session 9

1. CTL is defined by using the operators **A**, **E**, **X**, and **U** which in turn can be used in defining additional operators:

$\mathbf{EX}P$: $\neg\mathbf{AX}\neg P$

$\mathbf{AF}P$: $\mathbf{A}(\top\mathbf{U}P)$

$\mathbf{EF}P$: $\mathbf{E}(\top\mathbf{U}P)$

$\mathbf{AG}P$: $\neg\mathbf{EF}\neg P$

$\mathbf{EG}P$: $\neg\mathbf{AF}\neg P$

For each of these new operators, define its semantics as was done for the basic operators in the lecture notes. For example,

$\mathcal{M}, s \models \mathbf{AX}P$ if and only if $\mathcal{M}, t \models P$ for all t for which sRt holds.

2. LTL is defined by using the operators **X** and **U** which in turn can be used in defining additional operators:

$\mathbf{F}P$: $\top\mathbf{U}P$

$\mathbf{G}P$: $\neg\mathbf{F}\neg P$

$P\mathbf{R}Q$: $\neg\big((\neg P)\mathbf{U}(\neg Q)\big)$

For each of these new operators, define its semantics as was done for the basic operators in the lecture notes. For example,

$\mathcal{M}, x \models \mathbf{X}P$ if and only if $\mathcal{M}, x^1 \models P$.

3. Let P and Q be atomic formulas and $\mathcal{F} = \langle S, R\rangle$, where

$$S = \{s_0, s_1, s_2\}$$
$$R = \{\langle s_0, s_1\rangle, \langle s_1, s_2\rangle, \langle s_2, s_2\rangle\}.$$

Define a valuation v for P and Q in the worlds of frame $\mathcal{F}$ in such a way that

$$\mathcal{M}, x \models P\mathbf{U}Q \quad \text{and} \quad \mathcal{M}, x \not\models Q\mathbf{R}P.$$

hold for the full path $x = (s_0, s_1, s_2, s_2, s_2, \ldots)$ of the model $\mathcal{M} = \langle S, R, v\rangle$.

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Exercise Session 10

1. Formalize the following sentences in CTL.

a) P is true in the current state and Q is true in some state in the future.

b) P is true in some state in the future and, starting from that state, false in all states in the future.

c) If P is true in any two successive states, then starting from the latter of these two states there is a state in the future in which Q is true.

d) If P is true in the current state, then P stays true until Q becomes true. Otherwise, P must become true in at most two steps.

e) There is a path on which P is true until a state is reached starting from which Q alternates successively between true and false.

f) On a path, if there is a state in which P is true, then there is no state on the path in which Q or R is true.

2. a) Find a model in which the CTL formula $\mathbf{AF}P$ is satisfiable and the LTL formula $\mathbf{GF}P$ is unsatisfiable.

b) Find a model in which the CTL formula $\mathbf{EFAG}P$ is valid and the LTL formula $\mathbf{FG}P$ is not.

c) Find a model in which the LTL formula $\mathbf{FX}P$ is satisfiable and the CTL formula $\mathbf{EFAX}P$ is unsatisfiable.

3. Find a model in which the sets of CTL* formulas

a) $\big\{\mathbf{E}(\neg V \mathbf{U} P), \mathbf{E}(V \mathbf{U} \neg P), \mathbf{AF}(V \rightarrow \mathbf{AX}\neg V) \wedge \mathbf{EF}V\big\}$

b) $\big\{\mathbf{AG}(P \rightarrow \mathbf{F}V), \mathbf{AF}\big(P \wedge \mathbf{F}(\neg P \wedge \mathbf{X}P)\big), \mathbf{A}(\neg V \mathbf{U} V)\big\}$

are valid.