T-79.5101 **Spring 2008**
Advanced Course in Computational Logic
Exercise Session 1

1. Find a model for the following formulas.

a) $\neg\big((P \to Q) \to (Q \to P)\big)$

b) $\big((P \lor \neg R) \leftrightarrow R\big) \land (P \to Q)$

2. Does $\neg Q$ follow logically from the set of formulas

$$\Sigma = \{Q \to P, R \to (P \land Q), P \to (Q \land R)\}$$

If this is the case, construct an analytic tableau as a proof.

If not, give a counterexample.

3. Determine the conjunctive and disjunctive normal forms of the formula

$$(P \to Q) \to (P \lor Q).$$

4. Find a model for the following sentence.

a) $\exists x_1 \exists x_2 P(x_1, x_2) \land \forall x_1 \forall x_2 \big(P(x_1, x_2) \to P(x_2, x_1)\big)$

b) $\forall x_1 \exists x_2 P(x_1, x_2) \land \forall x_1 \forall x_2 \forall x_3 \big(P(x_1, x_2) \land P(x_2, x_3) \to P(x_1, x_3)\big)$

5. Prove the following sentences using the method of analytic tableaux.

a) $\big(\forall x P(x) \land \forall x Q(x)\big) \to \forall x \big(P(x) \lor Q(x)\big)$

b) $\exists y \big(\exists x P(x) \to P(y)\big)$

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Exercise Session 2

1. Denote the sentence 'agent knows that φ' by $K\varphi$. What is the meaning of the following formulas in natural language?

(a) $\varphi \to K\varphi$

(b) $\neg K\varphi \to K\neg K\varphi$

(c) $K(\varphi \to \psi) \to (K\varphi \to K\psi)$

(d) $K\varphi \lor K\neg\varphi$

2. Denote the sentences 'agent knows that φ' and 'φ is consistent with the agent's knowledge' by $K\varphi$ and $L\varphi$, respectively. Formalize the following sentences.

(a) If φ is true, then it is consistent with the agent's knowledge that agent knows that φ.

(b) If φ and ψ are consistent with the agent's knowledge, then $\varphi \land \psi$ is consistent with the agent's knowledge.

(c) If the agent knows that φ, then φ is consistent with the agent's knowledge.

(d) If it is consistent with the agent's knowledge that φ is consistent with the agent's knowledge, then φ is consistent with the agent's knowledge.

3. Formalize the following sentences in the language of modal logic.

(a) Agent A knows that agent B knows that it's raining, but agent B doesn't know that agent A knows that agent B knows that it is raining.

(b) Agent A knows that agent B doesn't know whether it's raining or not.

(c) Agent B knows that agent A knows whether it's raining or not.

(d) Agent A doesn't know whether agent B knows that agent knows that it's raining.

4. Let $\mathcal{M} = \langle S, R, v \rangle$ be a possible world model with

$$\begin{aligned} S &= \{s_1, s_2, s_3\}, \\ R &= \{\langle s_1, s_2 \rangle, \langle s_1, s_3 \rangle, \langle s_3, s_1 \rangle, \langle s_3, s_3 \rangle\}, \end{aligned}$$

$v(s_1, A) = v(s_2, B) = v(s_3, A) =$ true and $v(s, P) =$ false otherwise.

Which of the following claims hold?

(a) $\mathcal{M}, s_1 \Vdash \Box A$

(b) $\mathcal{M}, s_1 \Vdash \Diamond B \to \Box\Diamond\top$

(c) $\mathcal{M}, s_3 \Vdash \Diamond\Diamond\Box\bot$

(d) $\mathcal{M}, s_1 \Vdash \Box(B \vee \Box\Diamond A)$

(e) $\mathcal{M}, s_1 \Vdash \Diamond(\Box A \wedge \Box\neg A)$.

5. Let $\mathcal{M} = \langle S, R, v \rangle$ be a model with

$$\begin{aligned} S &= \{s_1, s_2, s_3, s_4, s_5\}, \\ R &= \{\langle s_1, s_1 \rangle, \langle s_1, s_2 \rangle, \langle s_1, s_3 \rangle, \langle s_1, s_4 \rangle, \langle s_2, s_3 \rangle, \langle s_3, s_5 \rangle, \langle s_4, s_1 \rangle, \\ &\quad \langle s_4, s_5 \rangle, \langle s_5, s_2 \rangle, \langle s_5, s_5 \rangle\}, \end{aligned}$$

$v(s_1, A) = v(s_4, A) = v(s_5, A) =$ true and $v(s, A) =$ false otherwise.

Find a world $s \in S$ in which

$$\mathcal{M}, s \Vdash \Box\Diamond\Box\Diamond A$$

holds.

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Exercise Session 3

1. Let A and B be atomic formulas. Show that the following formulas are not valid in all frames (give a counterexample for each formula).

a) $\Diamond A \to \Box A$

b) $\neg\Box A \to \Box\neg\Box A$

c) $\Diamond(\Diamond A \wedge \Box A) \to \Box\Diamond A$

d) $(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$

2. Show that the following holds for any formula A: $\Diamond\top$ is valid in a model if and only if $\Box A \to \Diamond A$ is valid in the model.

3. Let ϕ denote the formula

$$\Box((\Box\Box A \to \Diamond\Box A) \wedge \Diamond\Box(\Box A \to \Diamond A)) \to (\Diamond(\Diamond A \to \Box A) \to ((\Diamond A \wedge \Box\Diamond A) \vee \Diamond\Box\Box\neg A))$$

Furthermore, let it be known that ϕ is true in world s_4 of the model $\mathcal{M} = \langle S, R, v \rangle$, where

$$\begin{aligned} &S = \{s_1, s_2, s_3, s_4, s_5\}, \\ &R = \{\langle s_1, s_1 \rangle, \langle s_1, s_2 \rangle, \langle s_1, s_5 \rangle, \langle s_2, s_5 \rangle, \langle s_3, s_2 \rangle, \langle s_3, s_4 \rangle, \langle s_4, s_3 \rangle, \langle s_4, s_5 \rangle\} \end{aligned}$$

and $v(s_1, A) = v(s_2, A) = v(s_3, A) =$ true, and $v(s_4, A) = v(s_5, A) =$ false. Find a model $\mathcal{M}'$ with four possible worlds so that ϕ is true in some world of $\mathcal{M}'$.

4. Let $S = \{s_1, s_2, s_3, s_4\}$ and $R = \{\langle s_1, s_2 \rangle, \langle s_2, s_3 \rangle, \langle s_3, s_4 \rangle, \langle s_4, s_1 \rangle\}$. Find a frame $\langle S', R' \rangle$ which fulfills the following conditions:

(i) The frame $\langle S', R' \rangle$ has two possible worlds.

(ii) For all formulas P the following holds: If P is valid in $\langle S, R \rangle$, then P is valid in $\langle S', R' \rangle$.