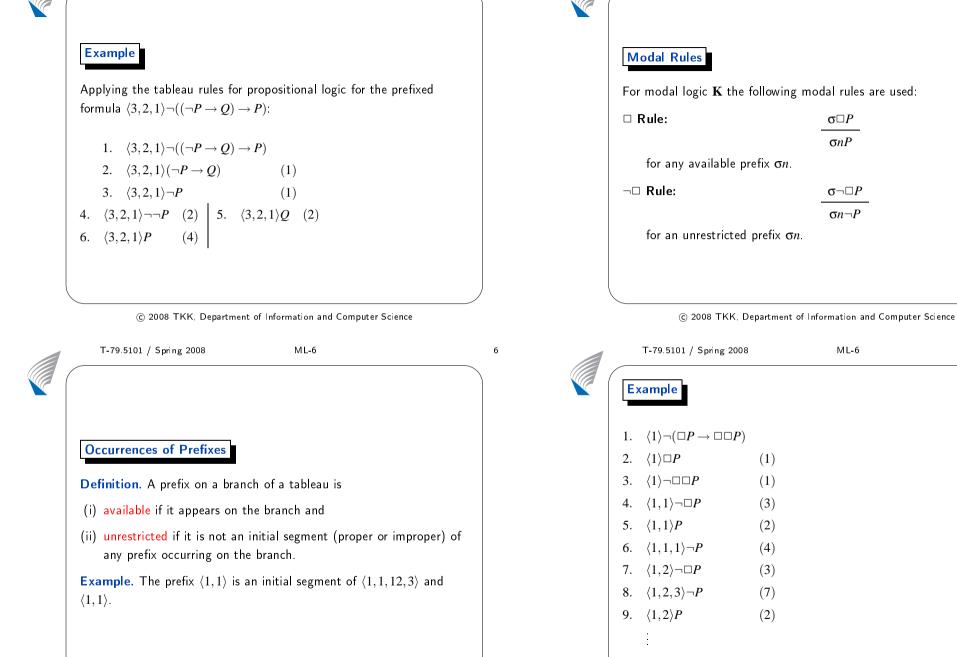
T-79.5101 / Spring 2008 ML-6	1	T-79.5101 / Spring 2008	ML-6	
TABLEAU METHOD FOR MODAL LOG	SICS		possible worlds using pro determine the accessibil	-
1. Tableau method for modal logic ${f K}$			pty finite sequence of na $1,1,1,1,111,2 angle$ are prefix	
 2. Soundness 3. Completeness 		prefix and P is a for		
 Other modal logics Logical consequence 		•	ula P is true in the world I formulas: $\langle 1 angle (P ee eg P),$	- /
M. Fitting: <i>Basic Modal Logic</i> , 1.9 (pp. 396 – 403).		-	efix which is obtained fro number <i>n.</i> For example,	
		•	σn is K -accessible from 11 K-accessible from $\langle 1$	
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1. Tableau Method of Modal Logic K • Motivation: Hilbert-style proofs are very hard to discow Applying the Modus Ponens rule: in order to derive Q , or to derive P and $P \rightarrow Q$ first. But what is an appropriate Example. Does $\{A \rightarrow B, A \rightarrow C\} \models A \rightarrow (B \land C)$ hold? 1. $A \rightarrow B$ 2. $A \rightarrow C$	ver. one needs		•	and prefixed formulas

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Tableau Proofs

Definition.

- A branch of a tableau is closed if it contains
 - 1. prefixed formulas σP and $\sigma \neg P$ for some formula *P* and prefix σ ; or
 - 2. a prefixed formula $\sigma\bot$ or a prefixed formula $\sigma\neg\top$ for some prefix $\sigma.$
- A tableau is closed if every branch in it is closed.
- A proof of a formula P is a tableau that has been constructed with the prefixed formula $\langle 1 \rangle \neg P$ as the root and that is closed.

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	Т-79.5101 / Sp	ing 200	8	Ν	1L-6
Ex	ample				
A t	ableau proof	for th	e form	nula $\Box(P \land Q)$	$(Q) \rightarrow (\Box P \land \Box Q)$:
	1. $\langle 1 \rangle \neg (\Box ($	$P \wedge Q$) → (□	$P \wedge \Box Q))$	
	2. $\langle 1 \rangle \Box (P /$	(Q)	,	,,	(1)
	3. $\langle 1 \rangle \neg (\Box H)$	$P \wedge \Box Q$	<u>?</u>)		(1)
4.	$\langle 1 \rangle \neg \Box P$	(3)	5.	$\langle 1 angle \neg \Box Q$	(3)
6.	$\langle 1,2 angle eg P$	(4)	10.	$\langle 1,3 angle eg Q$	(5)
7.	$\langle 1,2 \rangle P \wedge Q$	(2)	11.	$\langle 1,3\rangle P \wedge Q$	(2)
8.	$\langle 1,2 angle P$	(7)	12.	$\langle 1,3 \rangle P$	(11)
9.	$\langle 1,2 angle Q$	(7)	13.	$\langle 1,3 \rangle Q$	(11)
	×			×	

E>	kample			
A t	ableau pi	roof fo	or the formula $\top \leftrightarrow \Box \top$:	
	1.	$\langle 1 \rangle \neg ($	$\top \leftrightarrow \Box \top$)	
2.	$\langle 1 angle \neg \top$	(1)	$4. \langle 1 \rangle \top \qquad (1)$ $5. \langle 1 \rangle \neg \Box \top \qquad (1)$ $6. \langle 1, 2 \rangle \neg \top \qquad (5)$ \times	
3.	$\langle 1 \rangle \Box \top$	(1)	5. $\langle 1 \rangle \neg \Box \top$ (1)	
	×		6. $\langle 1,2 \rangle \neg \top$ (5)	
			×	

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(2. Soundness	
	Theorem . If there is a tableau proof for a formula P , then P is K -valid.	
	 Proof. (using the following steps) We define the concept of K-satisfiability for a tableau: a tableau is K-satisfiable if it has a branch that corresponds to a model in a particular way (to be defined below). 	
	 Then we show that (i) a K-satisfiable tableau cannot be closed. 	
	(ii) If a formula P is not K -valid, then the tableau consisting only of a root node $\langle 1 \rangle \neg P$ is K -satisfiable.	
	(iii) Every tableau rule preserve K-satisfiability: is a tableau is K-satisfiable before applying a rule, then it is K-satisfiable after applying the rule.	
	Hence, if P is not K -valid, the tableau for P remains open.	

K-Satisfiability of a Tableau

Definition.

- A tableau is K-satisfiable if one of its branches is K-satisfiable.
- A branch of a tableau is **K**-satisfiable if the set of prefixed formulas occurring on the branch is **K**-satisfiable.
- A set of prefixed formulas Σ is **K**-satisfiable if there is a model $\mathcal{M} = \langle S, R, \nu \rangle$ and a mapping \mathcal{N} from the prefixes appearing in Σ to the set S such that
 - 1. If prefixes σ and τ occur in the set Σ and τ is **K**-accessible from σ , then $\mathcal{N}(\sigma)R\mathcal{N}(\tau)$.
 - 2. If $\sigma P \in \Sigma$, then $\mathcal{M}, \mathcal{N}(\sigma) \parallel P$.

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A More Detailed Proof

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Based on the definition above we can show the following:

(i) A K-satisfiable tableau cannot be closed.

Assume that a **K**-satisfiable tableau is closed. By **K**-satisfiability the tableau has a **K**-satisfiable branch. Also this branch is closed, i.e., contains prefixed formulas σQ and $\sigma \neg Q$ (or $\sigma \bot$ or $\sigma \neg \top$). But this leads to a contradiction because for the branch there is a model \mathcal{M} and a mapping \mathcal{N} such that $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid -Q$ and

 $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \neg Q$ (or $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \bot$ or $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \neg \top$) which is impossible. Hence (i) holds.

A More Detailed Proof—cont'd

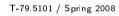
(ii) If a formula P is not **K**-valid, then the tableau containing only the root node $\langle 1 \rangle \neg P$ is **K**-satisfiable.

This holds because if the formula P is not **K**-valid, it has a counter-model $\mathcal{M} = \langle S, R, v \rangle$ such that for some world $s \in S$, $\mathcal{M}, s \mid \mid -\neg P$. Hence, the tableau containing only the root node and the branch $\{\langle 1 \rangle \neg P\}$ is **K**-satisfiable using the mapping $\mathcal{N}(\langle 1 \rangle) = s$.

(iii) Let a tableau Γ be **K**-satisfiable. Then it has a branch whose set of prefixed formulas Σ is **K**-satisfiable for some model $\mathcal{M} = \langle S, R, \nu \rangle$ and mapping \mathcal{N} .

We show that if a tableau rule is applied to $\Gamma,$ then the resulting tableau Γ' also K-satisfiable.

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Tableau Rules Preserve K-Satisfiability

Consider a tableau rule that is applied to a K-satisfiable branch (and let Σ be the set of prefixed formulas on the branch):

• σβ

$\sigma\beta_1 \,|\, \sigma\beta_2$

Then the tableau Γ' has two branches with the sets of prefixed formulas $\Sigma_1 = \Sigma \cup \{\sigma\beta_1\}$ and $\Sigma_2 = \Sigma \cup \{\sigma\beta_2\}$.

Since $\sigma\beta \in \Sigma$, $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \beta$,

and, thus, $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \beta_1$ or $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \beta_2$.

 $\implies \Gamma'$ is **K**-satisfiable.

σα

 $\sigma \alpha_1$

 $\sigma \alpha_2$

 $\sigma \Box P$

 σnP

formulas is $\Sigma_1 = \Sigma \cup \{\sigma nP\}$.

 $\mathcal{N}(\sigma)Rt \ \mathcal{M}, t \parallel P \text{ holds.}$

 $\implies \Gamma'$ on **K**-satisfiable.

formulas $\Sigma_1 = \Sigma \cup \{\sigma n \neg P\}$.

 $\implies \Gamma'$ on **K**-satisfiable.

such that $\mathcal{N}(\sigma)Rt$ and $\mathcal{M},t \mid\mid = \neg P$.

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• $\sigma \neg \Box P$ $\sigma n \neg P$

holds.

can be shown in a similar way as the resulting tableau contains the

Then the resulting tableau Γ' has a branch whose set of prefixed

As $\sigma \Box P \in \Sigma$, then $\mathcal{M}, \mathcal{N}(\sigma) \mid\mid = \Box P$, and, hence, for all t such that

Since σn occurs on the branch and is **K**-accessible from σ , then

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Then the resulting tableau Γ' has a branch whose set of prefixed

Since $\sigma \neg \Box P \in \Sigma$, $\mathcal{M}, \mathcal{N}(\sigma) \mid \mid = \neg \Box P$, and, thus, there is a world t

mapping \mathcal{N} can be extended: $\mathcal{N}(\sigma n) = t$. Then $\mathcal{M}, \mathcal{N}(\sigma n) \mid\mid = \neg P$

Because σn is not an initial segment of any prefix in Σ , the

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 $\mathcal{N}(\sigma)R\mathcal{N}(\sigma n)$ holds. Hence, $\mathcal{M}, \mathcal{N}(\sigma n) \parallel P$ holds.

for some available prefix σn .

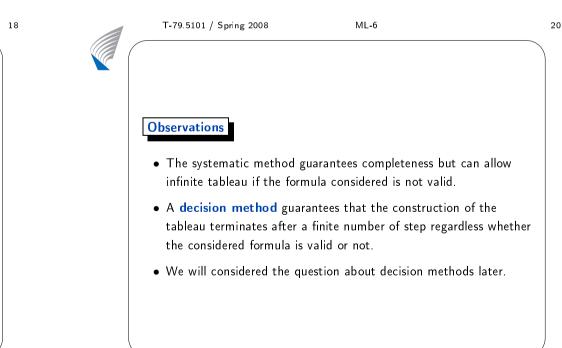
for some unrestricted prefix σn .

set of prefixed formulas $\Sigma_1 = \Sigma \cup \{\sigma \alpha_1, \sigma \alpha_2\}$.

3. Completeness

- How to guarantee that every valid formula has a tableau proof when the branches of a tableau can be infinite?
- When have the rules been applied fairly/enough times?
- For constructing tableaux a systematic method is needed where all the rules have been applied sufficiently, i.e., for every open branch θ it holds that
 - (i) If $\sigma \neg \neg P \in \theta$, then $\sigma P \in \theta$.
- (ii) If $\sigma\beta \in \theta$, then $\sigma\beta_1 \in \theta$ or $\sigma\beta_2 \in \theta$.
- (iii) If $\sigma \alpha \in \theta$, then $\sigma \alpha_1 \in \theta$ and $\sigma \alpha_2 \in \theta$.
- (iv) If $\sigma \neg \Box Q \in \theta$, then $\sigma n \neg Q \in \theta$, for some *n*.
- (v) If $\sigma \Box Q \in \theta$, $\sigma n Q \in \theta$ for all σn available on θ .

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Systematic K-Tableau for a Formula P:

- 1. Take as the root of the tableau $\langle 1 \rangle \neg P$.
- 2. Until the tableau is closed or all formulas have been marked used do:
- 2.1 Choose the top unused node σQ in the tableau.
- 2.2 If Q is not a literal, then for every open branch θ containing σQ do:
- If σQ is of the form $\sigma \neg \neg Q'$, extend θ by the node $\sigma Q'$.
- If σQ is of the form $\sigma \alpha$, extend θ by nodes $\sigma \alpha_1$ and $\sigma \alpha_2$.
- If σQ is of the form $\sigma\beta$, extend θ to two branches one containing $\sigma\beta_1$ and the other $\sigma\beta_2$.
- If σQ is of the form $\sigma \neg \Box P$, extend θ by $\sigma n \neg P$ for some prefix σn unrestricted in θ .
- If σQ is of the form $\sigma \Box P$, extend θ for all prefixes σn available on θ with σnP and then with the node $\sigma \Box P$.
- 2.3 Mark σQ used.

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Theorem. (Completeness) If a formula P is **K**-valid, then the systematic **K**-tableau for P will be closed.

Proof. We show that if the systematic **K**-tableau for *P* has an open branch, then *P* is not **K**-valid. Let θ be such a branch and $\mathcal{M} = \langle S, R, v \rangle$ a counter-model based on it:

- 1. S is the set of prefixes occurring in θ .
- 2. $\sigma R \tau$ iff τ is **K**-accessible from σ
- 3. $v(\sigma, Q) = \text{true iff } \sigma Q \text{ occurs on the branch } \theta \text{ for every atomic proposition } Q.$
- To prove the theorem it is enough to show the following lemma:

if $\sigma Q \in \theta$, then $\mathcal{M}, \sigma \parallel = Q$.

This implies the theorem because $\langle 1 \rangle \neg P$ occurs on every branch and, thus, $\mathcal{M}, \langle 1 \rangle \parallel \neg P$. This implies that P is not **K**-valid.

Induction Proof of the Lemma

We show for all prefixed formulas σQ that

if $\sigma Q \in \theta$, then $\mathcal{M}, \sigma \mid\mid = Q$.

Proof. This is done by induction on the length of the formula Q.

- (Q is an atomic proposition) If $\sigma Q \in \theta$, then $v(\sigma, Q) =$ true and $\mathcal{M}, \sigma \mid\mid -Q$.
- (*Q* is the negation of an atomic proposition) If $\sigma \neg Q' \in \theta$ and *Q'* is an atomic proposition, then $\sigma Q' \notin \theta$ and, hence, $\mathcal{M}, \sigma \models Q'$ and

 $\mathcal{M}, \sigma \mid\mid = \neg Q'.$

Induction hypothesis [IH]

If Q shorter that j and $\sigma Q \in \theta$, then $\mathcal{M}, \sigma \mid\mid = Q$.

Let the length of Q be j. Then Q is one of the following forms

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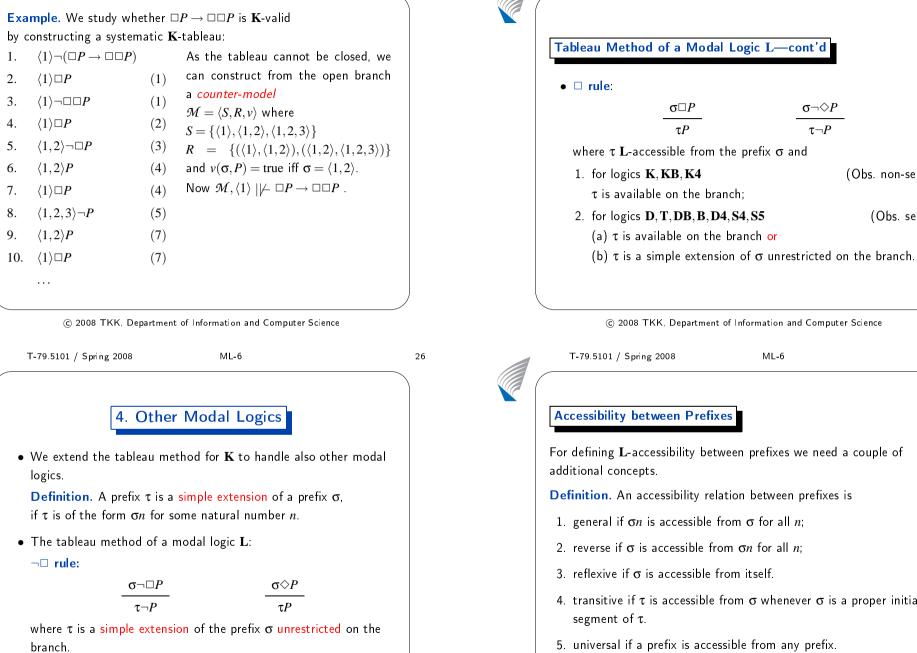
T-79.5101 / Spring 2008	ML-6	
• $(\neg \neg Q)$ If $\sigma \neg \neg Q \in \theta$, t By [IH] $\mathcal{M}, \sigma \mid\mid = Q$. H	~	
(i) I	then $\sigma\beta_1 \in \theta$ or $\sigma\beta_2 \in \theta$. r $\mathcal{M}, \sigma \mid\mid = \beta_2$. Thus, $\mathcal{M}, \sigma \mid$	– β.
	, then $\sigma lpha_1 \in heta$ and $\sigma lpha_2 \in heta$ nd $\mathcal{M}, \sigma \mid\mid = lpha_2$. Hence, \mathcal{M}	
• $(\neg \Box Q)$ If $\sigma \neg \Box Q \in \theta$, By [IH] $\mathcal{M}, \sigma n \mid \mid - \neg Q$	$\sigma n \neg Q \in heta$ for some n . 2. So $\mathcal{M}, \sigma \mid eq \Box Q$ as $\sigma R \sigma n$	
• $(\Box O)$ if $\sigma \Box O \subset A$ σm		

• $(\Box Q)$ If $\sigma \Box Q \in \theta$, $\sigma n Q \in \theta$ for all σn available on θ . By [IH] $\mathcal{M}, \sigma n \mid\mid = Q$. Hence, $\mathcal{M}, \sigma \mid\mid = \Box Q$.

So for every open branch θ , prefix σ and a formula Q: if $\sigma Q \in \theta$, then $\mathcal{M}, \sigma \mid\mid = Q$.

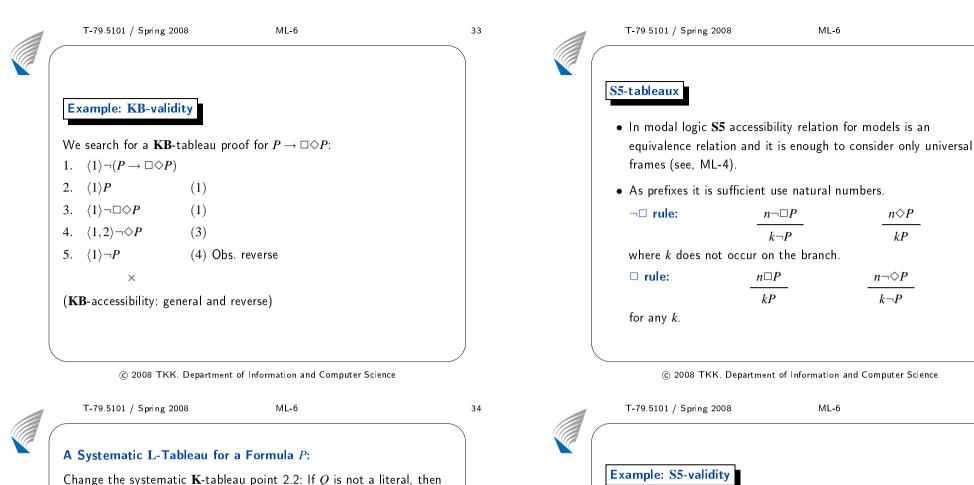
(Obs. non-serial)

(Obs. serial)



(c) 2008 TKK, Department of Information and Computer Science T-79 5101 / Spring 2008 ML-6 Accessibility between Prefixes For defining L-accessibility between prefixes we need a couple of **Definition.** An accessibility relation between prefixes is 1. general if σn is accessible from σ for all n; 2. reverse if σ is accessible from σn for all n: 3 reflexive if σ is accessible from itself 4. transitive if τ is accessible from σ whenever σ is a proper initial 5. universal if a prefix is accessible from any prefix.

L-Accessibility of Pre	fixes–cont'd			
L-Accessibility for some	logics L:		Example: T-validity	
$Logic\ \mathbf{L}$	L-accessibility		We construct a T -tableau proof for the formula	$a\Box P \rightarrow P$:
K , D	general		1. $\langle 1 \rangle \neg (\Box P \rightarrow P)$	
Т	general, reflexive		$2. \langle 1 \rangle \Box P \qquad (1)$	
KB,DB	general, reverse		$3. \langle 1 \rangle \neg P \tag{1}$	
В	general, reflexive, reverse		4. $\langle 1 \rangle P$ (2) Obs. reflexivity	
K4 , D4	general, transitive		×	
S 4	general, reflexive, transitive		(T -accessibility: general and reflexive)	
S5	universal			
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© 2008 TKK, Dep T-79.5101 / Spring 2008	partment of Information and Computer Science ML-6	30	© 2008 TKK, Department of Information and T-79.5101 / Spring 2008 ML-6	Computer Science
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T-79.5101 / Spring 2008 Example: D-validity	ML-6		T-79.5101 / Spring 2008 ML-6	
T-79.5101 / Spring 2008 Example: D-validity			T-79.5101 / Spring 2008 ML-6	
T-79.5101 / Spring 2008 Example: D-validity We search for a D -tablea	ML-6		T-79.5101 / Spring 2008 ML-6 Example: K4-validity We search for a K4 tableau proof for the formula	
T-79.5101 / Spring 2008 Example: D-validity We search for a D -tablea 1. $\langle 1 \rangle \neg (\Box P \rightarrow \neg \Box \neg P)$	ML-6 au proof for the formula $\Box P o eg \Box eg P$:		T-79.5101 / Spring 2008 ML-6 Example: K4-validity We search for a K4 tableau proof for the formula 1. $\langle 1 \rangle \neg (\Box P \rightarrow \Box \Box P)$	
Example: D-validity We search for a D -tablea 1. $\langle 1 \rangle \neg (\Box P \rightarrow \neg \Box \neg P)$ 2. $\langle 1 \rangle \Box P$	ML-6 au proof for the formula $\Box P ightarrow \neg \Box \neg P$: (1)		T-79.5101 / Spring 2008 ML-6 Example: K4-validity We search for a K4 tableau proof for the formulation $1. \langle 1 \rangle \neg (\Box P \rightarrow \Box \Box P)$ 2. $\langle 1 \rangle \Box P$ (1)	
Example: D -validity We search for a D -tablea 1. $\langle 1 \rangle \neg (\Box P \rightarrow \neg \Box \neg P)$ 2. $\langle 1 \rangle \Box P$ 3. $\langle 1 \rangle \neg \neg \Box \neg P$	ML-6 au proof for the formula $\Box P \rightarrow \neg \Box \neg P$: (1) (1)		T-79.5101 / Spring 2008 ML-6 Example: K4-validity We search for a K4 tableau proof for the formula 1. $\langle 1 \rangle \neg (\Box P \rightarrow \Box \Box P)$ 2. $\langle 1 \rangle \Box P$ (1) 3. $\langle 1 \rangle \neg \Box \Box P$ (1)	
Example: D-validity We search for a D -tablea 1. $\langle 1 \rangle \neg (\Box P \rightarrow \neg \Box \neg P)$ 2. $\langle 1 \rangle \Box P$ 3. $\langle 1 \rangle \neg \neg \Box \neg P$ 4. $\langle 1 \rangle \Box \neg P$ 5. $\langle 1, 2 \rangle \neg P$	ML-6 au proof for the formula $\Box P \rightarrow \neg \Box \neg P$: (1) (1) (3)		T-79.5101 / Spring 2008ML-6Example: K4-validityWe search for a K4 tableau proof for the formulation 1. $\langle 1 \rangle \neg (\Box P \rightarrow \Box \Box P)$ 1. $\langle 1 \rangle \neg (\Box P \rightarrow \Box \Box P)$ 2. $\langle 1 \rangle \Box P$ 2. $\langle 1 \rangle \Box P$ (1)3. $\langle 1 \rangle \neg \Box \Box P$ (1)4. $\langle 1, 2 \rangle \neg \Box P$ (3)	
Example: D -validity We search for a D -tablea 1. $\langle 1 \rangle \neg (\Box P \rightarrow \neg \Box \neg P)$ 2. $\langle 1 \rangle \Box P$ 3. $\langle 1 \rangle \neg \neg \Box \neg P$ 4. $\langle 1 \rangle \Box \neg P$ 5. $\langle 1, 2 \rangle \neg P$	ML-6 au proof for the formula $\Box P \rightarrow \neg \Box \neg P$: (1) (1) (3) (4) Observe: 2. (b)		T-79.5101 / Spring 2008 ML-6 Example: K4-validity We search for a K4 tableau proof for the formula 1. $\langle 1 \rangle \neg (\Box P \rightarrow \Box \Box P)$ 2. $\langle 1 \rangle \Box P$ (1) 3. $\langle 1 \rangle \neg \Box \Box P$ (1) 4. $\langle 1, 2 \rangle \neg \Box P$ (3) 5. $\langle 1, 2, 3 \rangle \neg P$ (4)	



We search of a S5-tableau proof for a $\neg \Box P \rightarrow \Box \neg \Box P$: (1)(1)(2)(3)

 $n \Diamond P$

kP

 $n \neg \Diamond P$

35

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1. $1\neg(\neg\Box P\rightarrow\Box\neg\Box P)$ 2. $1 \neg \Box P$ 3. $1 \neg \Box \neg \Box P$ 4. $2\neg P$ 5. $3\neg\neg\Box P$ 6. $3\Box P$ (5)7. 2P (6)Х

• If σQ is of the form $\sigma \neg \Box P$ ($\sigma \Diamond P$), extend θ by $\sigma n \neg P$ ($\sigma n P$) for

for every prefix σ' available on θ that is **L**-accessible from σ ,

for every prefix σ' available on θ that is **L**-accessible from σ .

extend θ by $\sigma' P$ ($\sigma' \neg P$). If no such prefix σ' exists, extend θ by σnP ($\sigma n\neg P$) for some σn unrestricted in θ . In both cases add to

extend θ by $\sigma' P$ ($\sigma' \neg P$) and finally by $\sigma \Box P$ ($\sigma \neg \Diamond P$);

for every open branch θ containing σQ :

some prefix σn unrestricted in θ .

• If σQ is of the form $\sigma \Box P$ ($\sigma \neg \Diamond P$),

(b) if L is one of D, DB, D4:

(a) if L is one of K. KB. K4. T. B. S4. S5:

the end of the branch $\theta \sigma \Box P (\sigma \neg \Diamond P)$.

Systematic S5-tableau for a Formula P:

- 1. Take as the root of the tableau $1\neg P$.
- 2. Until the tableau is closed or all nodes have been marked used do:
- 2.1 Choose the topmost unused node nQ.
- 2.2 If Q is not a literal, then for every open branch θ including nQ:
- If σQ is of the form $\sigma \neg \neg Q'$, extend θ by the node $\sigma Q'$.
- If nQ is of the form $n\alpha$, extend θ by $n\alpha_1$ and $n\alpha_2$.
- If nQ is of the form $n\beta$, extend θ to two branches one containing $n\beta_1$ and the other $n\beta_2$.
- If nQ is of the form n¬□P, extend θ by k¬P for some k unrestricted in θ and after this by kX for every j□X on the branch.
- If nQ is of the form $n\Box P$ extend θ by adding for all k available on θkP .
- 2.3 Mark *nQ* used.

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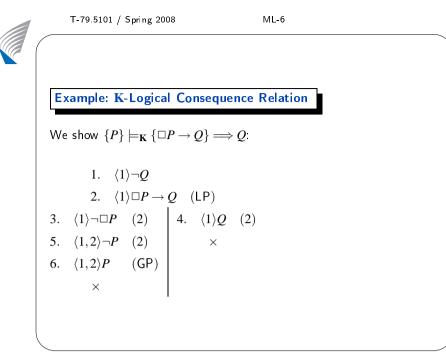
1	T-79.5101 / Spring 2008		ML-6
KI	045-Tableaux		
•	In modal logic KD45 i (see. ML-4):	t is sufficien	t to consider models of the form
	$M = \langle \{s_0\}$	$\cup S, \{\langle s,t\rangle \mid s$	$s \in \{s_0\} \cup S, t \in S\}, v\rangle.$
•	It is enough to use nat	tural numbe	rs as prefixes.
	¬□ rule:	$n \neg \Box P$	$n \diamondsuit P$
		$k \neg P$	kP
	where k does not occu	ir on the bra	nch.
	□ rule:	$n\Box P$	$n \neg \Diamond P$
		kP	$k \neg P$
	for any $k \neq 1$.		

Example: KI	045-Validity	
Compare the t	wo KD45 -table	aux:
$(\Box P \rightarrow P)$: 1.	$1\neg (\Box P \to P)$	
2.	$1\Box P$	(1)

3. $1 \neg P$ (1) $\Box (\Box P \rightarrow P): 1. \quad 1 \neg \Box (\Box P \rightarrow P)$ 2. $2 \neg (\Box P \rightarrow P)$ (1)
3. $2 \Box P$ (2)
4. $2 \neg P$ (2)
5. 2P (3) \times

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