T-79.5101 **kevät 2006**
Laskennallisen logiikan jatkokurssi
Laskuharjoitus 5
Ratkaisut

1. a)

1.	$\langle 1\rangle\neg\big(\Box P \to \Box(Q \to P)\big)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box(Q \to P)$	(1)
4.	$\langle 1,2\rangle\neg(Q \to P)$	(3)
5.	$\langle 1,2\rangle Q$	(4)
6.	$\langle 1,2\rangle\neg P$	(4)
7.	$\langle 1,2\rangle P$	(2)
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Box(P \to Q) \to (\neg\Box\neg P \to \neg\Box\neg Q)\big)$				
2.	$\langle 1\rangle\Box(P \to Q)$	(1)			
3.	$\langle 1\rangle\neg(\neg\Box\neg P \to \neg\Box\neg Q)$	(1)			
4.	$\langle 1\rangle\neg\Box\neg P$	(3)			
5.	$\langle 1\rangle\neg\neg\Box\neg Q$	(3)			
6.	$\langle 1\rangle\Box\neg Q$	(5)			
7.	$\langle 1,2\rangle\neg\neg P$	(4)			
8.	$\langle 1,2\rangle P$	(7)			
9.	$\langle 1,2\rangle\neg Q$	(6)			
10.	$\langle 1,2\rangle P \to Q$	(2)			
11.	$\langle 1,2\rangle\neg P$	(10)	12.	$\langle 1,2\rangle Q$	(10)
	$\otimes$			$\otimes$	

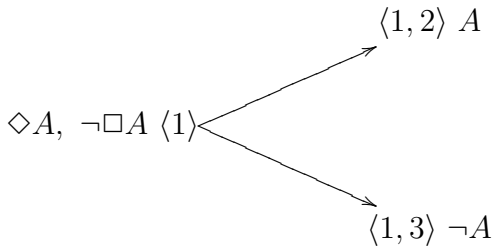
c)

1.	$\langle 1\rangle\neg\big((\Box P \wedge \Box Q) \to \Box(P \wedge Q)\big)$				
2.	$\langle 1\rangle\Box P \wedge \Box Q$	(1)			
3.	$\langle 1\rangle\neg\Box(P \wedge Q)$	(1)			
4.	$\langle 1\rangle\Box P$	(2)			
5.	$\langle 1\rangle\Box Q$	(2)			
6.	$\langle 1,2\rangle\neg(P \wedge Q)$	(3)			
7.	$\langle 1,2\rangle P$	(4)			
8.	$\langle 1,2\rangle Q$	(5)			
9.	$\langle 1,2\rangle\neg P$	(6)	10.	$\langle 1,2\rangle\neg Q$	(6)
	$\otimes$			$\otimes$	

2. a)

1.	$\langle 1\rangle\neg(\Diamond A \to \Box A)$	
2.	$\langle 1\rangle\Diamond A$	(1)
3.	$\langle 1\rangle\neg\Box A$	(1)
4.	$\langle 1,2\rangle\neg\neg A$	(2; $\Diamond$ on lyhennysmerkintä $\neg\Box\neg$:lle)
5.	$\langle 1,2\rangle A$	(4)
6.	$\langle 1,3\rangle\neg A$	(3)

Ei **K**-pätevä.



b)

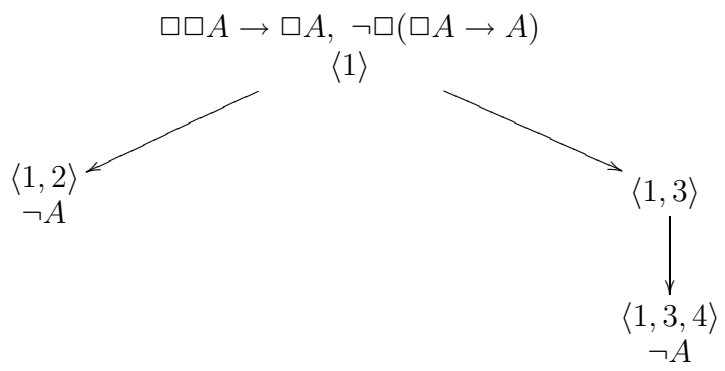
1.	$\langle 1\rangle\neg(\Diamond\Box A \vee \Box\Diamond\neg A)$	
2.	$\langle 1\rangle\neg\Diamond\Box A$	(1)
3.	$\langle 1\rangle\neg\Box\Diamond\neg A$	(1)
4.	$\langle 1\rangle\Box\neg\Box A$	(2)
5.	$\langle 1,2\rangle\neg\Diamond\neg A$	(3)
6.	$\langle 1,2\rangle\neg\Box A$	(4)
7.	$\langle 1,2\rangle\Box\neg\neg A$	(5)
8.	$\langle 1,2,3\rangle\neg A$	(6)
9.	$\langle 1,2,3\rangle\neg\neg A$	(7)
	$\otimes$	

Lause on **K**-pätevä.

c)

1.	$\langle 1\rangle\neg\big((\Box\Box A \to \Box A) \to \Box(\Box A \to A)\big)$				
2.	$\langle 1\rangle\Box\Box A \to \Box A$				(1)
3.	$\langle 1\rangle\neg\Box(\Box A \to A)$				(1)
4.	$\langle 1,2\rangle\neg(\Box A \to A)$				(3)
5.	$\langle 1,2\rangle\Box A$				(4)
6.	$\langle 1,2\rangle\neg A$				(4)
7.	$\langle 1\rangle\neg\Box\Box A$	(2)	8.	$\langle 1\rangle\Box A$	(2)
9.	$\langle 1,3\rangle\neg\Box A$	(7)	11.	$\langle 1,2\rangle A$	(8)
10.	$\langle 1,3,4\rangle\neg A$	(9)		$\otimes$	

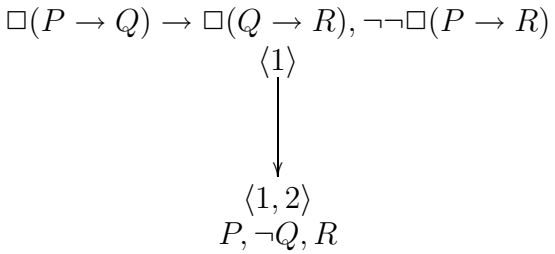
Ei **K**-pätevä.



3. a)

1.	$\langle 1\rangle\neg\Big(\big(\Box(P \to Q) \to \Box(Q \to R)\big) \to \neg\Box(P \to R)\Big)$					
2.	$\langle 1\rangle\Box(P \to Q) \to \Box(Q \to R)$				(1)	
3.	$\langle 1\rangle\neg\neg\Box(P \to R)$				(1)	
4.	$\langle 1\rangle\Box(P \to R)$				(3)	
5.	$\langle 1\rangle\neg\Box(P \to Q)$		(2)	6.	$\langle 1\rangle\Box(Q \to R)$	(2)
7.	$\langle 1,2\rangle\neg(P \to Q)$		(5)			
8.	$\langle 1,2\rangle P$		(7)			
9.	$\langle 1,2\rangle\neg Q$		(7)			
10.	$\langle 1,2\rangle P \to R$		(4)			
11.	$\langle 1,2\rangle\neg P$	(10)		12.	$\langle 1,2\rangle R$	(10)
	$\otimes$					

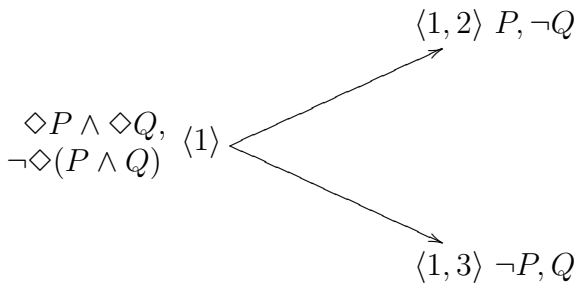
Ei **K**-pätevä.



b)

1.	$\langle 1\rangle\neg\big((\Diamond P \wedge \Diamond Q) \rightarrow \Diamond(P \wedge Q)\big)$						
2.	$\langle 1\rangle\Diamond P \wedge \Diamond Q$				(1)		
3.	$\langle 1\rangle\neg\Diamond(P \wedge Q)$				(1)		
4.	$\langle 1\rangle\Diamond P$				(2)		
5.	$\langle 1\rangle\Diamond Q$				(2)		
6.	$\langle 1\rangle\Box\neg(P \wedge Q)$				(3)		
7.	$\langle 1,2\rangle P$				(4)		
8.	$\langle 1,2\rangle\neg(P \wedge Q)$				(6)		
9.	$\langle 1,2\rangle\neg P$	(8)	10.	$\langle 1,2\rangle\neg Q$	(8)		
	$\otimes$		11.	$\langle 1,3\rangle Q$	(5)		
			12.	$\langle 1,3\rangle\neg(P \wedge Q)$	(6)		
			13.	$\langle 1,3\rangle\neg P$	(11)	13.	$\langle 1,3\rangle\neg Q$ (12)
							$\otimes$

Ei **K**-pätevä.



c)

1.	$\langle 1\rangle\neg\big(\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q)\big)$				
2.	$\langle 1\rangle\Box(P \wedge Q)$				(1)
3.	$\langle 1\rangle\neg(\Box P \wedge \Box Q)$				(1)
4.	$\langle 1\rangle\neg\Box P$	(3)	5.	$\langle 1\rangle\neg\Box Q$	(3)
6.	$\langle 1,2\rangle\neg P$	(4)	10.	$\langle 1,2\rangle\neg Q$	(5)
7.	$\langle 1,2\rangle P \wedge Q$	(2)	11.	$\langle 1,2\rangle P \wedge Q$	(2)
8.	$\langle 1,2\rangle P$	(7)	12.	$\langle 1,2\rangle P$	(11)
9.	$\langle 1,2\rangle Q$	(7)	13.	$\langle 1,2\rangle Q$	(11)
	$\otimes$			$\otimes$	