

Here is a small example to illustrate division of polynomials for homework 2, exercise 1b. The task was to use Extended Euclidean algorithm to compute inverse of $x^3 + x \pmod{x^5 + x^2 + x^1}$.

Now, the setup is $r_0 = x^5 + x^2 + 1$ and $r_1 = x^3 + x$, and first we are trying to solve q_2 and r_2 , so that $r_0 = q_2 \cdot r_1 + r_2$.

$$\begin{array}{r}
 x^3 + x \quad \left| \begin{array}{r}
 x^2 + 1 \\
 x^5 + x^3 \\
 \hline
 x^3 + x^2 + 0x + 1 \\
 x^3 + x \\
 \hline
 x^2 + x + 1
 \end{array}
 \right.
 \end{array}$$

As calculations are performed $\pmod{2}$, there is no need to separate addition and subtraction.

So, we get $q_2 = x^2 + 1$ and $r_2 = x^2 + x + 1$. This can be verified:
 $(x^2 + 1) \cdot (x^3 + x) + (x^2 + x + 1) = (x^5 + x^3 + x^3 + x) + (x^2 + x + 1) \equiv x^5 + x^2 + 1$.