1. Consider the three following finite state automata, where \( \Sigma_i = \{a, b\} \).

   **Automaton \( A_1 \):**

   ![Automaton A1](image1.png)

   **Automaton \( A_2 \):**

   ![Automaton A2](image2.png)

   **Automaton \( A_3 \):**

   ![Automaton A3](image3.png)

   (a) Construct the finite state automaton \( A_a = A_1 \cup A_2 \).
   
   (b) Construct the finite state automaton \( A_b = A_1 \cap A_2 \).
   
   (c) Is the language accepted by \( A_b \) non-empty? If not, give a word accepted by \( A_b \).
   
   (d) Complement the deterministic automaton \( A_1 \), and give the resulting automaton \( A_d \).
   
   (e) Give a deterministic finite state automaton \( A_e \), which accepts the same language as \( A_3 \).
   
   (f) Describe the language accepted by \( A_1 \) as a function of the number of occurrences of \( a \) and \( b \).