T–79.4301 **Autumn 2008**

Parallel and Distributed Systems

Tutorial 8 – Solutions

1.

$L: \quad \Sigma_L = \{a, b\}$ $\qquad L': \quad \Sigma_{L'} = \{a, b\}$

$L \leq_{sim} L'$: The relation $R = \{(s_0, t_0), (s_1, t_0)\}$ is a simulation relation between L and L' that contains the pair (s_0, t_0).

$L' \leq_{sim} L$: The relation $R' = \{(t_0, s_0)\}$ is a simulation relation between L' and L that contains the pair (t_0, s_0).

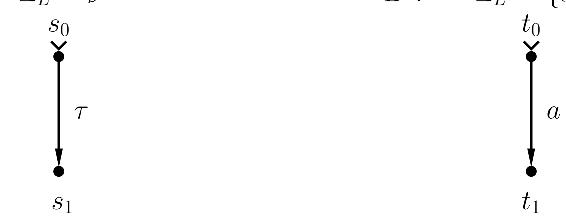
Assume that $L \sim L'$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1\} \times \{t_0\}$ such that $(s_0, t_0) \in B$ holds.

Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_0) \in B$.

Similarly, because $(s_1, t_0) \in B$ and $t_0 \xrightarrow{a} t_0$ hold, there exists a state $s \in \{s_0, s_1\}$ such that $s_1 \xrightarrow{a} s$ and $(s, t_0) \in B$ holds. This is, however, a contradiction, because there is no such state s in the LTS L. Therefore the LTSs L and L' are not bisimilar, and thus $L \not\sim L'$.

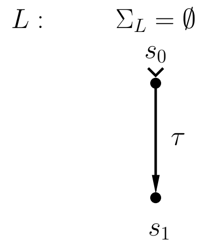
2.

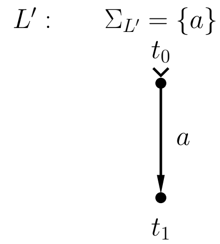
$L: \quad \Sigma_L = \emptyset$ $\qquad L': \quad \Sigma_{L'} = \{a\}$



Because $traces(L) = \{\varepsilon\} \subseteq \{\varepsilon, a\} = traces(L')$, $L \leq_{tr} L'$ holds, but because $traces(L') = \{\varepsilon, a\} \not\subseteq \{\varepsilon\} = traces(L)$, $L' \leq_{tr} L$ does not hold.

3.





Because $traces(L) = \{\varepsilon\} \subseteq \{\varepsilon, a\} = traces(L')$, $L \leq_{tr} L'$ holds.

Assume that $L \leq_{sim} L'$ holds, i.e., that there exists a simulation $R \subseteq \{s_0, s_1\} \times \{t_0, t_1\}$ such that $(s_0, t_0) \in R$ holds.

Because $(s_0, t_0) \in R$ and $s_0 \xrightarrow{\tau} s_1$ hold, then, because R is a simulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_0 \xrightarrow{\tau} t$ and $(s_1, t) \in R$ hold. This is a contradiction, because there is no such state in L', and thus $L \not\leq_{sim} L'$.

4. The claim "If L and L' are bisimilar, then both $L \leq_{sim} L'$ and $L' \leq_{sim} L$ hold" is true. If L and L' are bisimilar, there is a bisimulation B between them, and that relation satisfies the requirements of a simulation between L and L' in both directions.

5. $L \sim L'$ implies $traces(L) = traces(L')$, because

$$
\begin{aligned}
& L \sim L' \\
\Rightarrow\ & L \leq_{sim} L' \text{ and } L' \leq_{sim} L \\
\Rightarrow\ & L \leq_{tr} L' \text{ and } L' \leq_{tr} L \\
\Rightarrow\ & traces(L) \subseteq traces(L') \text{ and } traces(L') \subseteq traces(L) \\
\Rightarrow\ & traces(L) = traces(L').
\end{aligned}
$$