T–79.4301 **Autumn 2008**
Parallel and Distributed Systems
Tutorial 7 – Solutions

1. a) Assume that $L_1 \sim L_2$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1, s_2, s_3, s_4\} \times \{t_0, t_1, t_2, t_3\}$ such that $(s_0, t_0) \in B$ holds.

   Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0, t_1, t_2, t_3\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_1) \in B$.

   Similarly, because $(s_1, t_1) \in B$ and $t_1 \xrightarrow{c} t_3$ hold, there exists a state $s \in \{s_0, s_1, s_2, s_3, s_4\}$ such that $s_1 \xrightarrow{c} s$ and $(s, t_3) \in B$ hold. This is, however, a contradiction, because there is no such state s in the LTS L_1. Therefore our assumption that there exists a bisimulation B between L_1 and L_2 (with $(s_0, t_0) \in B$) is incorrect, and thus $L_1 \not\sim L_2$.

   b) Assume that $L_3 \sim L_4$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1, s_2\} \times \{t_0, t_1\}$ such that $(s_0, t_0) \in B$ holds.

   Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{\tau} s_1$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_0 \xrightarrow{\tau} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_1) \in B$.

   Because $(s_1, t_1) \in B$ and $s_1 \xrightarrow{\tau} s_2$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_1 \xrightarrow{\tau} t$ and $(s_2, t) \in B$ hold. This is again a contradiction, and therefore $L_3 \not\sim L_4$.

   c) The relation $B = \big\{(s_0, t_0), (s_1, t_0)\big\}$ is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_5 \sim L_6$ holds.

   d) The relation

$$B = \big\{(s_0, t_0), (s_1, t_1), (s_2, t_2), (s_2, t_3), (s_3, t_2), (s_3, t_3), (s_4, t_2)\big\}$$

   is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_7 \sim L_8$ holds.