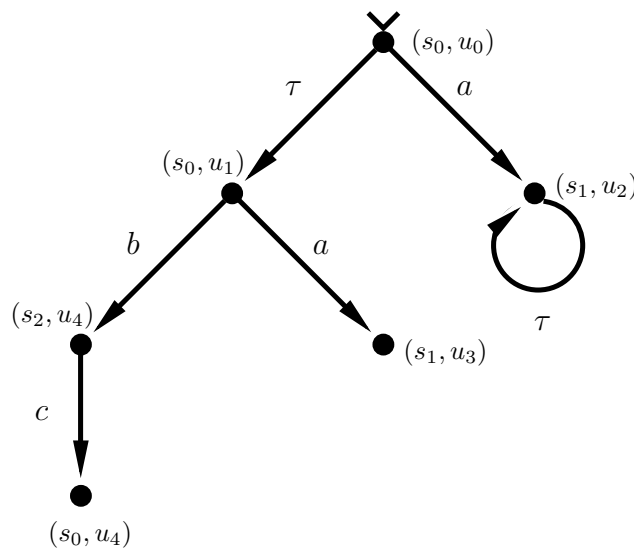


Parallel and Distributed Systems

Tutorial 6 – Solutions

1. a) Because the concepts of a conflict, a deadlock and a livelock are defined only with respect to reachable states, we focus on the reachable state space of the parallel composition of L_1 and L_3 (shown below).



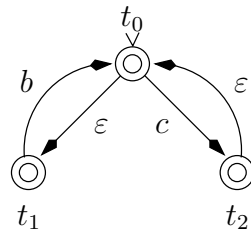
- b) The parallel composition of L_1 and L_3 contains two conflicts at states (s_0, u_0) and (s_0, u_1) , respectively:

$$\left((s_0, u_0), ((s_0, a, s_1), (u_0, a, u_2)), (-, (u_0, \tau, u_1)) \right), \text{ and} \\ \left((s_0, u_1), ((s_0, a, s_1), (u_1, a, u_3)), ((s_0, b, s_2), (u_1, b, u_4)) \right).$$

(In the first case, the LTS L_3 can choose between an a - and a τ -transition; in the second case, L_1 and L_3 can synchronise on either a or b .)

- c) The LTS $L_1 \parallel L_3$ contains deadlocks in states (s_0, u_4) and (s_1, u_3) , because these states have no successors.
- d) Because $(s_1, u_2) \xrightarrow{\tau} (s_1, u_2) \xrightarrow{\tau^*} (s_1, u_2)$ holds, the LTS $L_1 \parallel L_3$ has a livelock in state (s_1, u_2) .

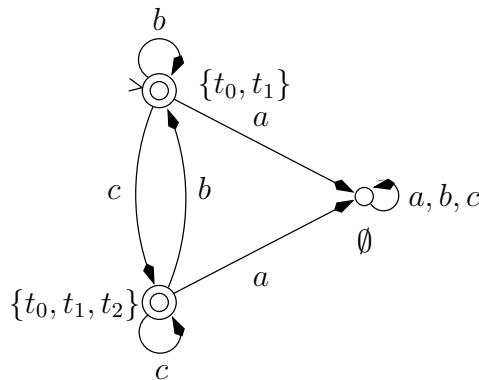
- e) Because neither L_1 nor L_3 “participates” in both of the global transitions $(-, (u_0, \tau, u_1))$ (starting from the state (s_0, u_0)) and $((s_2, c, s_0), -)$ (starting from the state (s_2, u_4)), these global transitions are independent.
- f) $traces(L_3) = \{\varepsilon, a, b\}$.
- g) $traces(L_1)$ is given by the regular expression $((bc)^*) \cup ((bc)^*a) \cup (b(cb)^*)$ (all words formed by tracing a path from s_0 to s_0 , from s_0 to s_1 , and from s_0 to s_2 , respectively).
- h) The LTS L_2 as an automaton with ε -transitions (all states are accepting):



Because t_0 is the only initial state of the automaton, the initial state of the corresponding deterministic automaton is

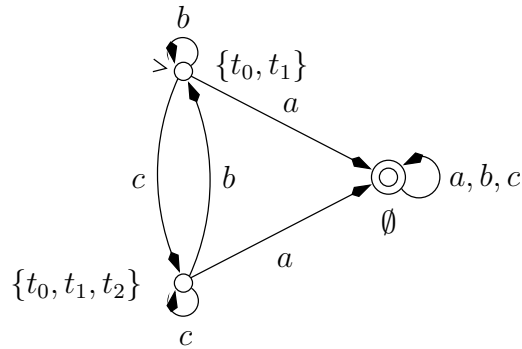
$$\{t \in \{t_0, t_1, t_2\} \mid t_0 \xrightarrow{\varepsilon^*} t\} = \{t_0, t_1\}.$$

The (reachable state space of) the deterministic automaton built from the above automaton is thus

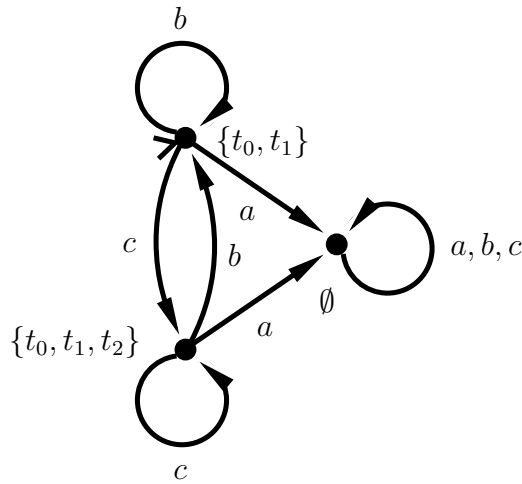


(note that to complement the automaton with respect to the alphabet $\Sigma_1 = \{a, b, c\}$, we have to use this alphabet already for determinisation, even though the alphabet of L_2 includes only the symbols b and c).

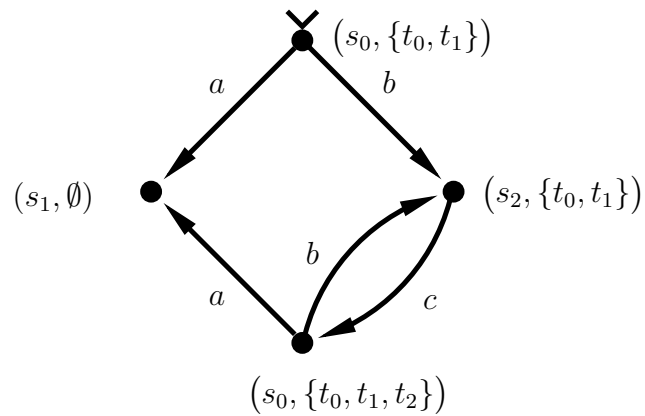
Swapping the final and non-final states yields the automaton



i) The automaton in h) as an LTS \bar{L}_2 :



The reachable part of the parallel composition of L_1 and \bar{L}_2 :



Because \emptyset is a final state of the automaton corresponding to the LTS \bar{L}_2 , and because the state (s_1, \emptyset) is reachable from the initial state of

$L_1 \parallel \bar{L}_2$, it follows that $traces(L_1) \subseteq traces(L_2)$ does *not* hold. A path from $(s_0, \{t_0, t_1\})$ to (s_1, \emptyset) can be used to find a trace of L_1 which is not a trace of L_2 : for example, $a \in traces(L_1) \setminus traces(L_2)$.