1. a) Assume that $L_1 \sim L_2$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1, s_2, s_3, s_4\} \times \{t_0, t_1, t_2, t_3\}$ such that $(s_0, t_0) \in B$ holds.

Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because $B$ is a bisimulation, there exists a state $t \in \{t_0, t_1, t_2, t_3\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_1) \in B$.

Similarly, because $(s_1, t_1) \in B$ and $s_1 \xrightarrow{c} s_2$ hold, there exists a state $s \in \{s_0, s_1, s_2, s_3, s_4\}$ such that $s_1 \xrightarrow{c} s$ and $(s, t_3) \in B$ hold. This is, however, a contradiction, because there is no such state $s$ in the LTS $L_1$. Therefore our assumption that there exists a bisimulation $B$ between $L_1$ and $L_2$ with $(s_0, t_0) \in B$ is incorrect, and thus $L_1 \not{\sim} L_2$.

b) Assume that $L_3 \sim L_4$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1, s_2\} \times \{t_0, t_1\}$ such that $(s_0, t_0) \in B$ holds.

Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because $B$ is a bisimulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_1) \in B$.

Because $(s_1, t_1) \in B$ and $s_1 \xrightarrow{c} s_2$ hold, then, because $B$ is a bisimulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_1 \xrightarrow{c} t$ and $(s_2, t) \in B$ hold. This is again a contradiction, and therefore $L_3 \not{\sim} L_4$.

c) The relation $B = \{(s_0, t_0), (s_1, t_0)\}$ is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_5 \sim L_6$ holds.

d) The relation $B = \{(s_0, t_0), (s_1, t_1), (s_2, t_2), (s_2, t_3), (s_3, t_2), (s_3, t_3), (s_4, t_2)\}$ is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_7 \sim L_8$ holds.

2.

For $L: \Sigma_L = \{a, b\}$ and $L': \Sigma_{L'} = \{a, b\}$,

\[
\begin{array}{c}
\text{a} \\
\text{a} \\
\text{s_0} \\
\text{b} \\
\text{s_1} \\
\text{b}
\end{array}
\]

\[
\begin{array}{c}
\text{t_0} \\
\text{a} \\
\text{b}
\end{array}
\]

is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_7 \sim L_8$ holds.
$L \leq_{\text{sim}} L'$: The relation $R = \{(s_0, t_0), (s_1, t_0)\}$ is a simulation relation between $L$ and $L'$ that contains the pair $(s_0, t_0)$.

$L' \leq_{\text{sim}} L$: The relation $R' = \{(t_0, s_0)\}$ is a simulation relation between $L'$ and $L$ that contains the pair $(t_0, s_0)$.

Assume that $L \sim L'$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1\} \times \{t_0\}$ such that $(s_0, t_0) \in B$ holds.

Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because $B$ is a bisimulation, there exists a state $t \in \{t_0\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold.

It follows that $(s_1, t_0) \in B$.

Similarly, because $(s_1, t_0) \in B$ and $t_0 \xrightarrow{a} t_0$ hold, there exists a state $s \in \{s_0, s_1\}$ such that $s_1 \xrightarrow{a} s$ and $(s, t_0) \in B$ holds. This is, however, a contradiction, because there is no such state $s$ in the LTS $L$. Therefore the LTSs $L$ and $L'$ are not bisimilar, and thus $L \not\sim L'$.

3. 

Because $\text{traces}(L) = \{\varepsilon\} \subseteq \{\varepsilon, a\} = \text{traces}(L')$, $L \leq_{\text{tr}} L'$ holds.

Assume that $L \leq_{\text{sim}} L'$ holds, i.e., that there exists a simulation $R \subseteq \{s_0, s_1\} \times \{t_0, t_1\}$ such that $(s_0, t_0) \in R$ holds.

Because $(s_0, t_0) \in R$ and $s_0 \xrightarrow{\tau} s_1$ hold, then, because $R$ is a simulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_0 \xrightarrow{\tau} t$ and $(s_1, t) \in R$ hold.

This is a contradiction, because there is no such state in $L'$, and thus $L \not\leq_{\text{sim}} L'$. 

$L': 
\begin{array}{c}
\varepsilon \\
\varepsilon \\
\end{array}$

$L: 
\begin{array}{c}
s_0 \\
\tau \\
s_1 \\
\end{array}$

\[ \Sigma_L = \emptyset \] 
\[ \Sigma_{L'} = \{a\} \]