1. Consider the three following finite state automata, where $\Sigma_i = \{a, b\}$.

   Automaton $A_1$:

   ![Automaton A1](image1)

   Automaton $A_2$:

   ![Automaton A2](image2)

   Automaton $A_3$:

   ![Automaton A3](image3)

   (a) Construct the finite state automaton $A_a = A_1 \cup A_2$.

   (b) Construct the finite state automaton $A_b = A_1 \cap A_2$.

   (c) Is the language accepted by $A_b$ non-empty? If not, give a word accepted by $A_b$.

   (d) Complement the deterministic automaton $A_1$, and give the resulting automaton $A_d$.

   (e) Give a deterministic finite state automaton $A_e$, which accepts the same language as $A_3$.

   (f) Describe the language accepted by $A_1$ as a function of the number of occurrences of $a$ and $b$. 