T–79.4301 **Spring 2006**
Parallel and Distributed Systems
Tutorial 7 – Solutions

1. a) Assume that $L_1 \sim L_2$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1, s_2, s_3, s_4\} \times \{t_0, t_1, t_2, t_3\}$ such that $(s_0, t_0) \in B$ holds.

   Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0, t_1, t_2, t_3\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_1) \in B$.

   Similarly, because $(s_1, t_1) \in B$ and $t_1 \xrightarrow{c} t_3$ hold, there exists a state $s \in \{s_0, s_1, s_2, s_3, s_4\}$ such that $s_1 \xrightarrow{c} s$ and $(s, t_3) \in B$ hold. This is, however, a contradiction, because there is no such state s in the LTS L_1. Therefore our assumption that there exists a bisimulation B between L_1 and L_2 (with $(s_0, t_0) \in B$) is incorrect, and thus $L_1 \not\sim L_2$.

   b) Assume that $L_3 \sim L_4$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1, s_2\} \times \{t_0, t_1\}$ such that $(s_0, t_0) \in B$ holds.

   Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{\tau} s_1$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_0 \xrightarrow{\tau} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_1) \in B$.

   Because $(s_1, t_1) \in B$ and $s_1 \xrightarrow{\tau} s_2$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_1 \xrightarrow{\tau} t$ and $(s_2, t) \in B$ hold. This is again a contradiction, and therefore $L_3 \not\sim L_4$.

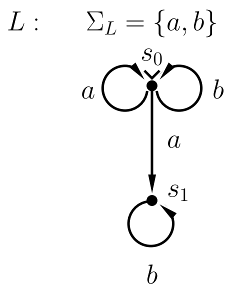
   c) The relation $B = \{(s_0, t_0), (s_1, t_0)\}$ is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_5 \sim L_6$ holds.

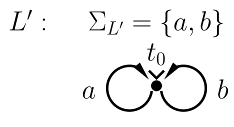
   d) The relation

$$B = \{(s_0, t_0), (s_1, t_1), (s_2, t_2), (s_2, t_3), (s_3, t_2), (s_3, t_3), (s_4, t_2)\}$$

   is a bisimulation such that $(s_0, t_0) \in B$ holds. Therefore $L_7 \sim L_8$ holds.

2.





$L \leq_{sim} L'$: The relation $R = \{(s_0, t_0), (s_1, t_0)\}$ is a simulation relation between L and L' that contains the pair (s_0, t_0).

$L' \leq_{sim} L$: The relation $R' = \{(t_0, s_0)\}$ is a simulation relation between L' and L that contains the pair (t_0, s_0).

Assume that $L \sim L'$ holds, i.e., that there exists a bisimulation $B \subseteq \{s_0, s_1\} \times \{t_0\}$ such that $(s_0, t_0) \in B$ holds.

Because $(s_0, t_0) \in B$ and $s_0 \xrightarrow{a} s_1$ hold, then, because B is a bisimulation, there exists a state $t \in \{t_0\}$ such that $t_0 \xrightarrow{a} t$ and $(s_1, t) \in B$ hold. It follows that $(s_1, t_0) \in B$.

Similarly, because $(s_1, t_0) \in B$ and $t_0 \xrightarrow{a} t_0$ hold, there exists a state $s \in \{s_0, s_1\}$ such that $s_1 \xrightarrow{a} s$ and $(s, t_0) \in B$ holds. This is, however, a contradiction, because there is no such state s in the LTS L. Therefore the LTSs L and L' are not bisimilar, and thus $L \not\sim L'$.

3.

Because $traces(L) = \emptyset \subseteq \{a\} = traces(L')$, $L \leq_{tr} L'$ holds.

Assume that $L \leq_{sim} L'$ holds, i.e., that there exists a simulation $R \subseteq \{s_0, s_1\} \times \{t_0, t_1\}$ such that $(s_0, t_0) \in R$ holds.

Because $(s_0, t_0) \in R$ and $s_0 \xrightarrow{\tau} s_1$ hold, then, because R is a simulation, there exists a state $t \in \{t_0, t_1\}$ such that $t_0 \xrightarrow{\tau} t$ and $(s_1, t) \in R$ hold. This is a contradiction, because there is no such state in L', and thus $L \not\leq_{sim} L'$.