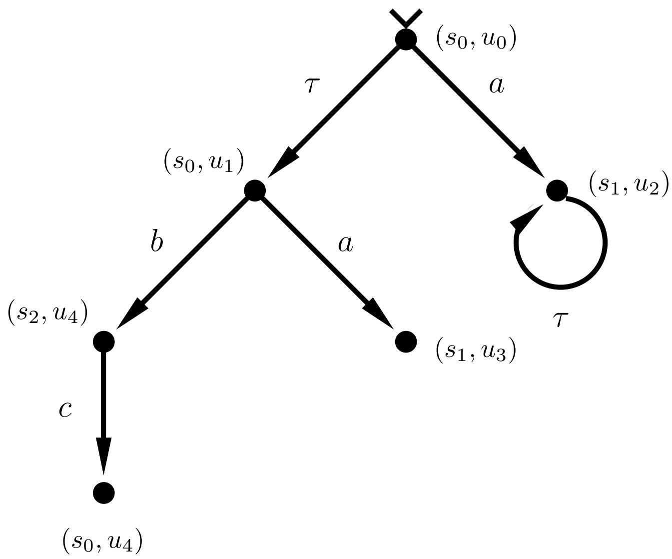
T–79.4301 **Spring 2006**
Parallel and Distributed Systems
Tutorial 6 – Solutions

1. a) Because the concepts of a conflict, a deadlock and a livelock are defined only with respect to reachable states, we focus on the reachable state space of the parallel composition of L_1 and L_3 (shown below).



b) The parallel composition of L_1 and L_3 contains two conflicts at states (s_0, u_0) and (s_0, u_1), respectively:

$$\big((s_0, u_0), ((s_0, a, s_1), (u_0, a, u_2)), (-, (u_0, \tau, u_1))\big), \text{ and}$$
$$\big((s_0, u_1), ((s_0, a, s_1), (u_1, a, u_3)), ((s_0, b, s_2), (u_1, b, u_4))\big).$$

(In the first case, the LTS L_3 can choose between an a- and a τ-transition; in the second case, L_1 and L_3 can synchronise on either a or b.)

c) The LTS $L_1 \,||\, L_3$ contains deadlocks in states (s_0, u_4) and (s_1, u_3), because these states have no successors.

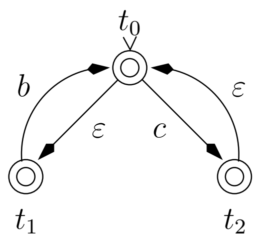
d) Because $(s_1, u_2) \xrightarrow{\tau} (s_1, u_2) \xrightarrow{\tau^*} (s_1, u_2)$ holds, the LTS $L_1 \,||\, L_3$ has a livelock in state (s_1, u_2).

e) Because neither L_1 nor L_3 "participates" in both of the global transitions $\big(-, (u_0, \tau, u_1)\big)$ (starting from the state (s_0, u_0)) and $\big((s_2, c, s_0), -\big)$ (starting from the state (s_2, u_4)), these global transitions are independent.

f) $\mathit{traces}(L_3) = \{a, b\}$.

g) $\mathit{traces}(L_1)$ is given by the regular expression $\big((bc)^*\big) \cup \big((bc)^* a\big) \cup \big(b(cb)^*\big)$ (all words formed by tracing a path from s_0 to s_0, from s_0 to s_1, and from s_0 to s_2, respectively).

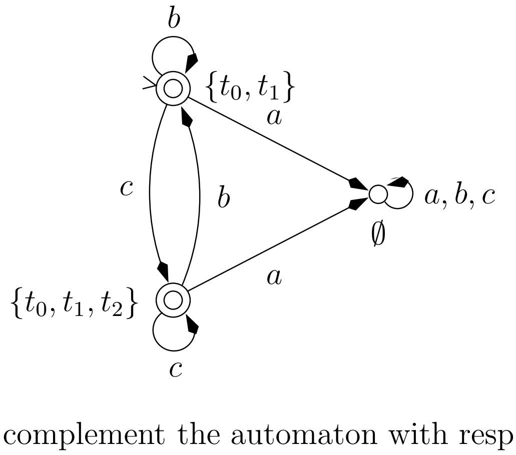
h) The LTS L_2 as an automaton with ε-transitions (all states are accepting):



Because t_0 is the only initial state of the automaton, the initial state of the corresponding deterministic automaton is

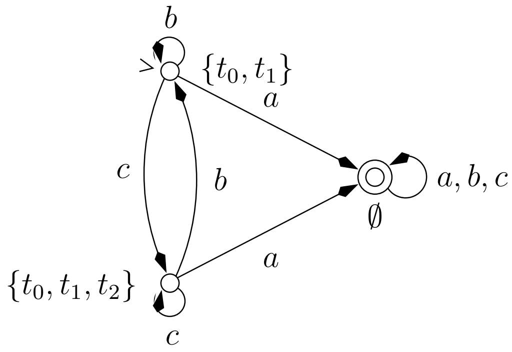
$$\big\{t \in \{t_0, t_1, t_2\} \,\big|\, t_0 \xrightarrow{\varepsilon^*} t\big\} = \{t_0, t_1\}.$$

The (reachable state space of) the deterministic automaton built from the above automaton is thus

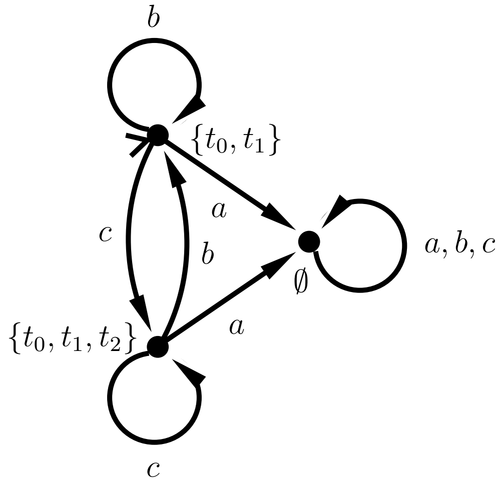


(note that to complement the automaton with respect to the alphabet $\Sigma_1 = \{a, b, c\}$, we have to use this alphabet already for determinisation, even though the alphabet of L_2 includes only the symbols a and b).

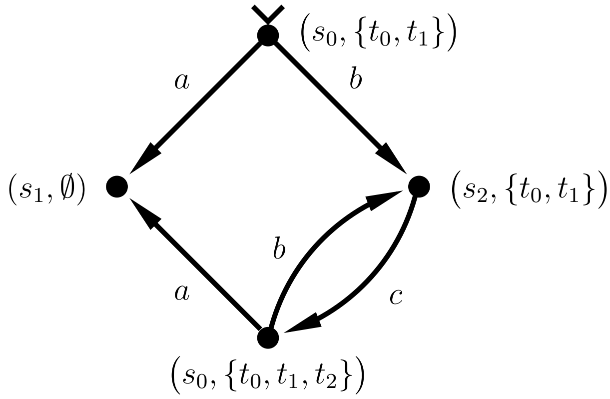
Swapping the final and non-final states yields the automaton



i) The automaton in h) as an LTS $\overline{L}_2$:



The parallel composition of L_1 and $\overline{L}_2$:



Because $\emptyset$ is a final state of the automaton corresponding to the LTS $\overline{L}_2$, and because the state $(s_1, \emptyset)$ is reachable from the initial state of

$L_1 \,||\, \overline{L}_2$, it follows that $\mathit{traces}(L_1) \subseteq \mathit{traces}(L_2)$ does *not* hold. A path from $\big(s_0, \{t_0, t_1\}\big)$ to $(s_1, \emptyset)$ can be used to find a trace of L_1 which is not a trace of L_2: for example, $a \in \mathit{traces}(L_1) \setminus \mathit{traces}(L_2)$.