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T-79.4201 Search Problems and Algorithms (4 cr)
Exam 20 Dec 2007, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and student number
- The text: "T-79.4201 Search Problems and Algorithms 20.12.2007"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Consider the following NP-complete DOMINATING SET problem:

   INSTANCE: Undirected graph $G = (V, E)$.
   QUESTION: Find a minimal size subset of nodes (vertices) of G, $D \subseteq V$, such that each node in V either is a member of D or has at least one neighbour in D, i.e. if $u \in V \setminus D$, then there is an edge $\{u, v\} \in E$ such that $v \in D$.

   Present in pseudocode form a Record-to-Record Travel approach to finding small dominating sets in a given input graph G. Describe particularly clearly: (a) what are the candidate solutions considered by your method and what is their neighbourhood relation, (b) what is the cost (objective) function associated to the candidate solutions, (c) how does one choose the next solution for consideration from the neighbourhood of a given candidate solution, and (d) how does one choose the initial candidate solution for the computation. (*Note:* Remember to take care of the condition that the solution eventually returned by your method really *is* a dominating set.)

2. a) Write the corresponding equivalent propositional formula for the Boolean circuit given as the following system of equations:

$$\begin{aligned} y_1 &= \mathsf{or}(y_2, y_3) \\ y_2 &= \mathsf{and}(y_4, y_5) \\ y_3 &= \mathsf{or}(y_4, y_6) \\ y_4 &= \mathsf{and}(y_7, y_8) \end{aligned}$$

   b) Describe a typical Solve procedure which is guaranteed to find a solution to a constraint satisfaction problem (CSP) whenever such a solution exists. Give a detailed enough description how the procedure works and simulate it to find a solution for the CSP

$$\langle C_1(x_1, x_2), C_2(x_1, x_2); x_1 \in \{1, 2, 3\}, x_2 \in \{1, 2, 3\}\rangle$$

   where $C_1 = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$ and $C_2 = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$.

3. a) Express the condition "if $x_1 + 5 < 0$, then $x_2 + 15 \leq 0$" as a set of linear constraints when $-100 \leq x_1, x_2 \leq 100$.

   b) Consider the following linear programming problem:

$$\begin{aligned} &\max\ x_1 + 2x_2 \text{ s.t.} \\ &x_1 - 5x_2 \geq 7 \\ &-x_1 + x_2 \leq -3 \\ &x_2 \geq 0 \end{aligned}$$

   Transform the problem into the standard form and give a basic feasible solution for the problem in the standard form and the value of the objective function in the basic feasible solution.

4. Outline an ant colony optimisation (ACO) scheme for the task of finding small dominating sets in graphs such as in problem 1 above. Specify in particular: (a) what are in your optimisation scheme the nodes and edges comprising the search trails of the "ants", (b) what are the start and end nodes of each trail, (c) how to assess the "quality" of a complete search trail that determines its pheromone reward, and (d) how the reward is distributed among the nodes and/or edges along the trail.

Grading: Each problem 10p, total 40p.