

Search Problems and Algorithms

T-79.4201

Ilkka Niemelä & Pekka Orponen

Laboratory for Theoretical Computer Science, TKK

Spring 2006

T-79.4201 Search Problems and Algorithms (4 ECTS)

“An introduction to the fundamental concepts, techniques and tools used in dealing with large, weakly structured combinatorial search spaces.”

Required course in the new A2-level Study Module in TCS.

Why this course?

- ▶ With the increase in computing power, continually new computation-intensive application areas emerge (e.g. various types of planning & scheduling, data mining, bioinformatics, ...)
- ▶ Many immediate problems in these areas are both computationally demanding & mathematically weakly structured (“Here is my messy objective function. Find a near-optimal solution to it – quickly!”)

Practical arrangements

Lecture 1: Overview of the course

- ▶ In such “quick-and-dirty” settings a search problem formulation is often the most effective (if not the only) approach.

Practical arrangements

Lectures: Thu 10-12 TB353, alternately by Ilkka Niemelä and Pekka Orponen

Tutorials: Fri 10–12 TB353, Antti Rusanen

Registration: by TOPI

Prerequisites: Basic knowledge of problem representations and logic, facility in programming, data structures and algorithms

Requirements: Examination (15 May) and three small programming assignments (announced 10 Feb, 24 Feb, 24 Mar, each due in three weeks)

Course home page:

<http://www.tcs.hut.fi/Studies/T-79.4201/>

Grading scheme: Details TBA, programming assignments pass/fail

Material

No existing textbook: lectures cover a wide range of material from several textbooks & current scientific literature.

Course problems based on lecture slides; uploaded to the course web site each week after lecture.

Examples of reference material:

- ▶ Aarts & Lenstra (Eds.), *Local Search in Combinatorial Optimization*. Wiley 1997.
- ▶ Hoos & Stützle, *Stochastic Local Search: Foundations and Applications*. Morgan Kaufmann 2005.

1. Overview of the Course

1.1. A Motivating Example

Twelve slightly different types of billets, numbered 1 ... 12, arrive for processing at a factory workshop. The workshop has four machines, numbered I ... IV, and four workers, named A ... D, who have different qualifications for working on the billets. To make things more complicated, there are also four specialised tools, numbered i ... iv, that are needed for processing the various billets. The requirements of machines, tools, and workers for the billets are indicated in the following table:

	Machine		Tool		Worker
I:	1 5 9	i:	1 2 3	A:	1 7 8
II:	2 6 10	ii:	4 9 10	B:	2 3 4
III:	3 7 11	iii:	5 11 12	C:	5 6 12
IV:	4 8 12	iv:	6 7 8	D:	9 10 11

Let's say processing each billet by a combination of the appropriate machine, tool & worker requires 1 hour. Any given machine, tool, or worker can only work on one billet at a time. Since there are 12 billets and 4 machines (as well as tools & workers), processing all the billets requires at least 3 hours. Can it be done in this minimal time?

How would you approach the preceding problem:

- (a) By hand? (Design an appropriate schedule!)
- (b) By computer, assuming that an arbitrary list of requirements such as above would be given as input? (The numbers of machines, tools, and workers do not need to be the same: this is just a peculiarity of the present example.)

Think about this problem; it will be discussed at next week's tutorial. You do not need to write any program code, but try to think about how you would approach task (b) of minimising the completion time for a given list of requirements.

Lecture 2: Combinatorial search and optimisation problems

I.N. 26 Jan

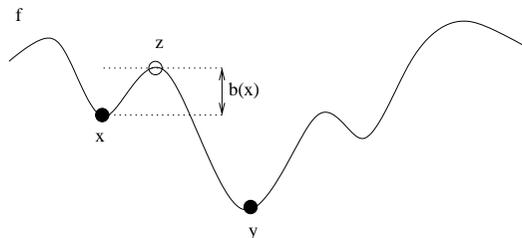
Common mathematical patterns in combinatorial search and optimisation: Satisfiability, Clique, Graph Colouring, Traveling Salesman, Set Cover.

Different types of problems and reductions between them.

Lecture 3: Search spaces and objective functions. Complete vs. local search

P.O. 2 Feb

Search spaces as “fitness landscapes”. Neighbourhoods and local search. Iterated local search. Simulated annealing. Backtrack search. A*/IDA*.



Lecture 4: Further local search techniques

P.O.. 9 Feb

Tabu search, Record-to-Record Travel. Local search methods for satisfiability. WalkSAT. Instructions for the 1st programming assignment.

Lecture 5: Constraint satisfaction: formalisms and modelling

I.N. 16 Feb

General representation of search problems as systems of constraints (e.g. propositional formulas)

$$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_4) \wedge (x_2 \vee \bar{x}_3 \vee x_4)$$

Case studies of translations.

**Lecture 6 : Constraint satisfaction: algorithms**

I.N. 23 Feb

The DPLL procedure. Other methods. WalkSAT revisited. Software tools for constraint satisfaction. Instructions for the 2nd programming assignment.

**Lecture 7: Linear and integer programming: formalism**

I.N. 16 Mar

General representation of problems as systems of linear equations over reals and integers.

$$\begin{aligned} \max \quad & F_\lambda := (1 - \lambda) \frac{1}{n} \sum_{i \in S} w_i r_i + \lambda \ell \\ \text{s.t.} \quad & \sum_{j \in V} f_{1j} = 0, \\ & \sum_{j \in V} f_{ij} = r_i + \sum_{j \in V} f_{ji}, & i \in S, \\ & \sum_{j \in V} f_{ij} = \sum_{j \in V} f_{ji}, & i \in R, \\ & \sum_{j \in V} T d_{ij} f_{ij} + \sum_{j \in V} T c_{ji} f_{ji} \leq e_i, & i \in V, \\ & r_i \leq s_i, & i \in S, \\ & w_i r_i \geq \ell, & i \in S, \\ & f_{ij} \geq 0, & i, j \in V, \\ & f_{ji} = 0. & i \in V. \end{aligned}$$

**Lecture 8: Linear and integer programming: modelling and tools**

I.N. 23 Mar

Case studies of problem translations. Software packages. Instructions for the 3rd programming assignment.



Lecture 9: Linear and integer programming: algorithms

I.N. 30 Mar

Branch & Bound methods. Overview of the simplex algorithm.

Lecture 10: Novel methods 2

P.O. 20 Apr

Ant algorithms. Belief and survey propagation.

Lecture 9: Novel methods 1

P.O.. 6 Apr

Genetic and other evolutionary algorithms.

Lecture 11: Special topics

P.O. 27 Apr

Properties of search runtime distributions. Phase transitions in local search. Computational complexity.

Lecture 12: Review and discussion of the course material

I.N. & P.O. 4 May