T-79.4201 **Spring 2006**
Search Problems and Algorithms
Tutorial 11, 28 April
Problems

1. Outline an Ant Colony Optimisation scheme for the 3-SAT problem. (Thus, the input is a propositional 3-cnf formula, and the goal is to find a truth assignment to the formula's variables that satisfies as many of its clauses as possible.)

2. Outline a Coevolutionary Genetic Algorithm for minimising the size of a Boolean circuit. (Thus, the input is a Boolean circuit, and the goal is to find an equivalent cicruit with as few gates as possible.)

3. Study the proof of the "No Free Lunch" theorem in Appendix A of D. H. Wolpert & W. G. Macready, "No Free Lunch Theorems for Optimization", *IEEE Transactions on Evolutionary Computation 1 (1997)*, 67–82. (The paper is accessible online via IEEE's Xplore service at `http://dx.doi.org/10.1109/4235.585893`, and a print copy of the journal can be found e.g. in the TCS lab lounge A357.) Extend the proof to the case when the algorithm a is stochastic, i.e. for a given sample $d \in \mathcal{D}$ determines only a distribution over the points $x \in \mathcal{X} - d^x$.

4. One corollary of the NFL theorem is that the expected value of any performance measure $\Phi(d_m^y)$ is independent of the optimisation algorithm a used, when the underlying objective function f is chosen uniformly at random from the space $\mathcal{Y}^{\mathcal{X}}$. To illustrate this result, compute explicitly the expected maximum value (i.e. $E[\max\{d^y(1), \ldots, d^y(m)\}]$) encountered in:

   (a) a local search of length $m = 2$ in the space of binary strings of length 2 ($\mathcal{X} = \{0,1\}^2$), when the range of the objective functions is $\mathcal{Y} = \{0,1\}$;

   (b) a local search of length $m = 3$ in the space of binary strings of length 3 ($\mathcal{X} = \{0,1\}^3$), when the range of the objective functions is $\mathcal{Y} = \{0,1,2\}$.

   (You do not need to verify that the expected maxima really are algorithm independent, but you might want to think about how the proof of the NFL theorem works in these special cases.)