

## Search Problems and Algorithms

## Tutorial 4, 17 February

## Problems

1. Consider the simulated annealing approach to solving the MAX CUT problem as discussed in problem 3 of last week's tutorial. What kind of cooling schedules would the simulated annealing convergence theorem presented at this week's lecture suggest in the case of a 3-regular input graph with  $n$  nodes? (3-regular  $\equiv$  every node has exactly 3 neighbours.)
2. Consider the relationship between branch-and-bound optimisation and the A\* algorithm. Reformulate the branch-and-bound approach to solving the TSP problem discussed at last week's lecture as an A\* graph search. What are the nodes, edges and edge costs of the search graph? What are the functions  $f$ ,  $g$  and  $h$  used in the A\* algorithm in this case?
3. Prove that if the search graph  $(X, N)$  is finite, then an A\* search using an admissible heuristic  $h$  always terminates with an optimal (i.e. minimum length) path from the start node  $x_0$  to some goal node  $x^* \in X^*$ . (*Hint*: Show that until the algorithm terminates, there is always some node  $x \in X$  in OPEN with the property that  $x$  lies on some optimal start-to-goal path and  $f(x) \leq f^*$ , where  $f^*$  is the cost of an optimal path.)
4. Draw the search space corresponding to the 3-SAT formula

$$(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$$

as a cube, and mark down at the corners of this cube the values of the objective function indicating the number of unsatisfied clauses at each point (= truth assignment). Trace the progress of a WalkSAT search with  $p = 0$  along the corners of the cube, starting at initial point  $(x_1, x_2, x_3) = (0, 0, 0)$ .