1. Outline an Ant Colony Optimisation scheme for the 3-SAT problem. (Thus, the input is a propositional 3-cnf formula, and the goal is to find a truth assignment to the formula’s variables that satisfies as many of its clauses as possible.)

2. Consider the Belief Propagation Algorithm applied to the 2-SAT formula

\[(x_1 \lor \bar{x}_2) \land (x_2 \lor x_3) \land (x_2 \lor \bar{x}_4)\]

(a) Draw the factor graph representation of the formula.

(b) List all the satisfying truth assignments explicitly, and based on this list compute the biases \(\beta_i(\xi) = \Pr_{x \in SAT}(x_i = \xi)\) for each of the variables \(x_i\) and values \(\xi \in \{0, 1\}\).

(c) Apply the Belief Propagation Algorithm to estimate the biases. (Note that in a tree-like factor graph such as here, the algorithm should converge in a single two-way pass from the leaves of the tree to a chosen root and back.)

3. Outline a belief propagation method for the Graph 3-Colourability problem discussed earlier in the lectures (i.e. Graph Colouring with the number of colours fixed to \(k = 3\)).