T-79.4201 **Autumn 2006**
Search Problems and Algorithms
Tutorial 5, 19 October
Problems

1. Give all solutions to the following constraint satisfaction problem (CSP)

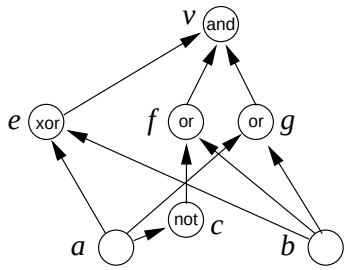
$$\langle \{C_1(z,y), C_1(y,x), C_1(x,z)\};$$
$$x \in \{1,2,3\}, y \in \{1,2,3\}, z \in \{1,2,3\}\rangle$$

   where $C_1 = \{(1,3),(1,2),(1,1),(2,3),(2,2),(3,3)\}$

2. Encode the SET COVER problem as a constraint satisfaction problem (CSP).

3. Encode the TSP optimization problem as a constrained optimization problem.

4. a) Give a propositional formula that expresses the Boolean function that the circuit below computes.



   b) Give a propositional formula in CNF that expresses the Boolean function

   i) $odd(x_1, x_2, x_3)$ which evaluates to true iff an odd number of x_1, x_2, x_3 have the value true;

   ii) $atleast_2(x_1, \ldots, x_n)$ which evaluates to true iff the number of $x_1, \ldots, x_n$ having the value true is at least 2;

   iii) $atmost_{n-1}(x_1, \ldots, x_n)$ which evaluates to true iff the number of $x_1, \ldots, x_n$ having the value true is at most $n-1$;

5. Give a Boolean circuit that performs the lexicographic comparison of two n bit strings, i.e., construct a circuit that has input gates $x_1, \ldots, x_n$, $y_1, \ldots, y_n$ and its output gate has the value true in a truth assignment T iff the bit string $b_1 b_2 \cdots b_n$ given as input for the gates $x_1, \ldots, x_n$ in T is lexicographically properly greater than $c_1 c_2 \cdots c_n$ given as input for the gates $y_1, \ldots, y_n$.

   Here when a bit string $b_1 b_2 \cdots b_n$ is given as input for the gates $x_1, \ldots, x_n$ in a truth assignment T it means that for $i = 1, \ldots, n$, if $b_i = 1$ then $T(x_i) = true$ else $T(x_i) = false$ and similarly for the input gates $y_1, \ldots, y_n$.

   Hint: For example, the bit string 01000 is lexicographically properly greater than 00111.