1. Give all solutions to the following constraint satisfaction problem (CSP)

\[ \{C_1(z, y), C_1(y, x), C_1(x, z)\}; x \in \{1, 2, 3\}, y \in \{1, 2, 3\}, z \in \{1, 2, 3\} \]

where \( C_1 = \{(1, 3), (1, 2), (1, 1), (2, 3), (2, 2), (3, 3)\} \)

2. Encode the SET COVER problem as a constraint satisfaction problem (CSP).

3. Encode the TSP optimization problem as a constrained optimization problem.

4. a) Give a propositional formula that expresses the Boolean function that the circuit below computes.

![Boolean Circuit Diagram]

b) Give a propositional formula in CNF that expresses the Boolean function

i) \( \text{odd}(x_1, x_2, x_3) \) which evaluates to true iff an odd number of \( x_1, x_2, x_3 \) have the value true;

ii) \( \text{atleast}_2(x_1, \ldots, x_n) \) which evaluates to true iff the number of \( x_1, \ldots, x_n \) having the value true is at least 2;

iii) \( \text{atmost}_{n-1}(x_1, \ldots, x_n) \) which evaluates to true iff the number of \( x_1, \ldots, x_n \) having the value true is at most \( n - 1 \);

5. Give a Boolean circuit that performs the lexicographic comparison of two \( n \) bit strings, i.e., construct a circuit that has input gates \( x_1, \ldots, x_n, y_1, \ldots, y_n \) and its output gate has the value true in a truth assignment \( T \) iff the bit string \( b_1b_2\cdots b_n \) given as input for the gates \( x_1, \ldots, x_n \) in \( T \) is lexicographically properly greater than \( c_1c_2\cdots c_n \) given as input for the gates \( y_1, \ldots, y_n \).

Here when a bit string \( b_1b_2\cdots b_n \) is given as input for the gates \( x_1, \ldots, x_n \) in a truth assignment \( T \) it means that for \( i = 1, \ldots, n \), if \( b_i = 1 \) then \( T(x_i) = true \) else \( T(x_i) = false \) and similarly for the input gates \( y_1, \ldots, y_n \).

Hint: For example, the bit string 01000 is lexicographically properly greater than 00111.