Quantum Gates

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Logical AND

$$00 \to 0$$
$$01 \to 0$$
$$10 \to 0$$
$$11 \to 1$$

Logical XOR

$$00 \to 0$$
$$01 \to 1$$
$$10 \to 1$$
$$11 \to 0$$

$$
\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |00\rangle \\
|10\rangle &\rightarrow |00\rangle \\
|11\rangle &\rightarrow |01\rangle
\end{aligned}
$$

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |00\rangle$$
$$|10\rangle \rightarrow |00\rangle$$
$$|11\rangle \rightarrow |01\rangle$$

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |00\rangle$$
$$|10\rangle \rightarrow |00\rangle$$
$$|11\rangle \rightarrow |01\rangle$$

$$
\begin{aligned}
|00\rangle &\to |a0\rangle \\
|01\rangle &\to |b0\rangle \\
|10\rangle &\to |c0\rangle \\
|11\rangle &\to |01\rangle
\end{aligned}
$$

Unitary transformation preserves orthogonality.

Unitary transformations do not destroy information.

Quantum gates must be *reversible*.

$$
\begin{aligned}
|000\rangle &\rightarrow |000\rangle \\
|010\rangle &\rightarrow |010\rangle \\
|100\rangle &\rightarrow |100\rangle \\
|110\rangle &\rightarrow |111\rangle
\end{aligned}
$$

$$U_{AND}|x_1, x_2, y\rangle = |x_1, x_2, y \oplus (x_1 \wedge x_2)\rangle$$

$$\begin{aligned} |001\rangle &\rightarrow |001\rangle \\ |011\rangle &\rightarrow |011\rangle \\ |101\rangle &\rightarrow |101\rangle \\ |111\rangle &\rightarrow |110\rangle \end{aligned}$$

Unitary transformations are linear:

$$U_{AND}(\alpha|s\rangle + \beta|t\rangle) = \alpha(U_{AND}|s\rangle) + \beta(U_{AND}|t\rangle)$$

Photons

Photons

Ions

Photons

Ions

Superconducting loops

Why bother?

Avoids von Neumann - Landauer limit.

Hadamard gate

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$H_A \otimes H_B \otimes I_C(|0\rangle_A|0\rangle_B|0\rangle_C)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$= \frac{1}{2}(|000\rangle + |010\rangle + |100\rangle + |110\rangle$$

$$U_{AND}\frac{1}{2}(|000\rangle \otimes |010\rangle \otimes |100\rangle \otimes |110\rangle)$$

$$= \frac{1}{2}(|000\rangle + |010\rangle + |100\rangle + |111\rangle)$$

$$U_{AND}\frac{1}{2}(|000\rangle \otimes |010\rangle \otimes |100\rangle \otimes |110\rangle)$$

$$= \frac{1}{2}(|000\rangle + |010\rangle + |100\rangle + |111\rangle)$$

Superpositions for parallel computation.

$$\psi \to U_N U_{N-1} \ldots U_1 \psi = U_\sigma \psi$$

Search from an unsorted database:

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Classical computing: $O(N)$

Search from an unsorted database:

Classical computing: $O(N)$

Quantum computer: $O(\sqrt{N})$

“All computing machines operating with the laws of [given] realm of physics are equivalent.”

-Gui Lu Long

Babbage's engine, Intel Core 2 Duo

equivalent

Babbage's engine, Intel Core 2 Duo

equivalent

Quantum computers (ions, photons, . . .)

equivalent

Where is particle wave duality?

Classical computers

Quantum particle computer

Duality quantum computer

New gates for dubits:

Wave dividers / splitters and combiners.

$$\psi \to \begin{array}{l} \nearrow p_1\psi \to p_1 U_1\psi \searrow \\ \searrow p_2\psi \to p_2 U_2\psi \nearrow \end{array} (p_1 U_1 + p_2 U_2)\psi$$

$$\psi \to (\sum p_i U_i)\psi$$

$$\psi \rightarrow \begin{array}{l} \nearrow p_1\psi \rightarrow p_1 U_1 \psi \searrow \\ \searrow p_2\psi \rightarrow p_2 U_2 \psi \nearrow \end{array} (p_1 U_1 + p_2 U_2)\psi$$

$$\psi \rightarrow (\sum p_i U_i)\psi$$

Photons - nonlinear quantum optics

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Giant molecules

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Classical computing: $O(N)$

Quantum *particle* computer: $O(\sqrt{N})$

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Duality quantum computer: Single query

Questions?

Thank you