Quantum Teleportation, Chapter 3.3

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Outline

1. Introduction
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2. Teleportation process
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    - Teleportation
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No-Cloning theorem

- No-Cloning theorem crucial for preventing Eve from accessing bits sent by Alice
- Causes problems for Alice and Bob too:
    - Photon detectors are not perfect and will occasionally produce false detections.
    - Photon passing through channel like optical fiber has some probability of being degraded or destroyed.
    - So even if a small percentage of Alice's photons manages to make through the long fiber, it gets impossible for Bob to distinguish Alice's photons from the background noise.
    - If photons could be cloned, one could place cloning devices at regular intervals along the fibre but this is not possible.

Quantum teleportation

- One solution to the problem is quantum teleportation.
    - If we can arrange two perfectly entangled particles, one at location A and other at B, a state of a third photon arriving at A can be teleported directly to B.
    - By repeating this process, it is possible in principle to extend key distribution scheme to arbitrary distance.
    - Though arranging a well-separated entangled pair is difficult.

Bell measurement

- Four mutually orthogonal two-qubit states, each of which is maximally entangled:

$$\left|\Phi^+\right\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$$

$$\left|\Phi^-\right\rangle = (1/\sqrt{2})(|00\rangle - |11\rangle)$$

$$\left|\Psi^+\right\rangle = (1/\sqrt{2})(|01\rangle + |10\rangle)$$

$$\left|\Psi^-\right\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$$

- This is called Bell measurement, after John S. Bell

Staging

- Let us assume that we have particles a and b at locations A (Alice) and B (Bob) and they are in entangled state $|\Phi^+\rangle$.
- At some point particle c arrives at location A whose state $|s\rangle = \alpha|0\rangle + \beta|1\rangle$ is to be teleported to B.
- The combined state of the particles c, a and b, where cab is the order in each term, can be written as:

$$|s\rangle \otimes |\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

Teleportation, step 1

- Alice performs a Bell measurement to subsystem consisting of particles a and c.
- To obtain result of this measurement we use rule for measurements on subsystems defined on previous lecture and reexpress the above state in terms of the outcome vectors of the measurement.
- No physical change is done yet.

Teleportation, step 1

$$\begin{aligned}|s\rangle \otimes |\Phi^+\rangle &= \frac{1}{2}\Big[|\Phi^+\rangle \otimes \Big(\alpha|0\rangle + \beta|1\rangle\Big)\\ &+|\Phi^-\rangle \otimes \Big(\alpha|0\rangle - \beta|1\rangle\Big)\\ &+|\Psi^+\rangle \otimes \Big(\beta|0\rangle + \alpha|1\rangle\Big)\\ &+|\Psi^-\rangle \otimes \Big(-\beta|0\rangle + \alpha|1\rangle\Big)\Big]\end{aligned}$$

- Alice makes the Bell measurement on particles a and c. Above equation shows us the state of the particle b after the measurement.
- For example, if Alice gets the outcome $|\Phi^+\rangle$, the state of the particle b is $\alpha|0\rangle + \beta|1\rangle$.

Teleportation, step 2

- Alice sends to Bob the outcome of her measurement through a classical channel.
- Bob performs a unitary transformation to particle b. Transformation depends on the result of Alice's measurement:

Outcome	Transformation
$\lvert\Phi^+\rangle$	I
$\lvert\Phi^-\rangle$	Z
$\lvert\Psi^+\rangle$	X
$\lvert\Psi^-\rangle$	XZ

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Teleportation, step 2

- Once Bob has performed the appropriate transformation, particle b is guaranteed to be in the state $|s\rangle$.
- Example: Alice gets the outcome $|\Psi^+\rangle$ so Bob's particle is in state $\beta|0\rangle + \alpha|1\rangle$. Bob applies the transformation X:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = |s\rangle$$

Teleportation, observations

- State $|s\rangle$ was not cloned, the states of the a and c particles were collapsed in the measurement.
- Classical communication from Alice to Bob contains no information about the state of $|s\rangle$.
- Probabilities of the four outcomes of Alice's measurement are all equal to $1/4$ regardless of values α and β.
- For example, probability of outcome $|\Phi^{+}\rangle$:

The probability of outcome $|m_i\rangle$ is $p_i = \langle v_i | v_i \rangle$

$$\alpha^2+\beta^2 = 1 \Rightarrow ((1/2)\alpha)^2+((1/2)\beta)^2 = (1/4)\alpha^2+(1/4)(1-\alpha^2) = 1/4$$

Teleportation, observations

- Teleportation is not instantaneous. Bob needs the signal from Alice to tell him the outcome of her measurement. Without it he has nothing useful.

Teleportation in practice

- Creating a perfectly entangled pair is difficult.
- Optical fibers are imperfect. The pair will not be anymore perfectly entangled when the other member has been sent over the fiber.
- Entanglement purification. Can be done in principle as long as the channel is not too noisy.
- So again the imperfectness of the fiber sets us limitations on transmission distance.

Teleportation in practice

- However we may chain multiple entangled pairs spanning some shorter distance to achieve arbitrary transmission distances.
- And probably use Bennett-Brassard key distribution scheme to prevent Eve from accessing any of the "middle links".
- Currently (book published in 2006) we don’t have the technology to do entanglement purification on large scale.

Summary

- In principle, quantum teleportation can be used to extend quantum key distribution scheme to arbitrary distances.
- Some classical channel is still needed for signaling the outcome of the measurement and probably also for arranging a perfectly entangled pair.
- Teleportation has been successfully demonstrated in a number of experiments but entanglement purification is not yet possible on a large scale.