Routing

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OUTLINE

① Routing definition
② Protocols
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  ➔ distance vector
  ➔ shortest-path spanning tree
  ➔ sparser gossip
  ➔ min-hop routing
③ Summary
Routing

Entity $x$ wants to communicate with entity $y$.

Broadcast is waste of resources.

Better – choose single path in the network from $x$ to $y$. Message is relayed only through entities on the path.

Routing: The process of determining path between two entities.
Routing

Definitions

- cost of link – depends on system (time delay, transmission cost, link reliability), also known as length
- cost of path – sum of individual link costs
- shortest path – path with minimum cost

Shortest-path routing: The process of determining the most economic path between two entities
Each entity has information how to reach other entities in the network. This information is stored in the routing table.

We are interested in construction of the tables and keeping them up to date.

Assumed restrictions are IR

- bidirectional links (BL)
- connectivity (CN)
- total reliability (TR)
- initial distinct values (ID)
Consider adjacency matrix of the network.

Each entity knows its own row of the matrix.

The entities exchange smaller pieces of information by *gossiping* with each other.
Every entity broadcasts its own information and after receiving every other entity’s information builds shortest-path routing table locally.

Each entity builds entire map of the network. Not very good performance. Requires a lot of storage for large network $O(n^2)$. Messages sent $O(nm)$. 
Iterative approach using distance vector protocol.

Initially entity $x$ knows only distance to its direct neighbors. All other nodes are at distance $\infty$.

In each iteration, entity communicates its distance vector with its direct neighbors. (no broadcasting!)

Entity updates its information if neighbors have found better routes.
Iterative Construction

When $x$ receives information that its direct neighbor $y$ can reach $z$ with certain cost. Then $x$ uses the route through $y$ if its total cost is less than using the previous route $x$ had to $z$.

The protocol converges to the correct routing table after at most $n - 1$ iterations.

The amount of storage required is proportional to the routing table $O(n)$, not to entire map. But the actual number of sent messages is larger $O(n^2m)$, and in worst case $O(n^4)$. 
**Shortest-path spanning tree**

Target: without increasing storage requirement, construct routing table with less sent messages.

Full paths of routing table form a spanning tree.

We just need to construct shortest-path spanning tree starting from root entity $s$. 
We are given connected sub-graph so far $T$.

Consider all the links directly outside of $T$. Assign to each such link $(x, y)$ the cost of going from $s$ to $y$.

Each time add the node $y$ with minimum cost to $T$ and start over.

How to do this in a distributed way?
SHORTEST-PATH SPANNING TREE

Operation
① root $s$ broadcasts in $T$ that iteration starts
② each entity in $T$ computes locally the best outgoing link
③ overall minimum is found by using distributed minimum finding protocol
④ root notifies the chosen new node that it is part of $T$

Continue until all nodes are in $T$. 
However, this only calculates routing table for the root $s$. Needs to be repeated for each individual entity.

Total messages sent for all entities is then $O(n^3)$. Better than distance vector, but still too much.

Note! A lot is repeated when neighbors $x$ and $y$ are calculating their spanning trees. The shortest path from $x$ to $y$ is the same as from $y$ to $x$ but reversed. Determining the other is enough.
SPARSER GOSSIP

Constructing a sparser graph gives improvements when simulating gossiping.

① divide network into separate spanning subgraphs
② elect leader in each subgraph
③ connect each pair of subgraph leaders together
④ each leader executes gossiping for each node in its subset
⑤ whenever message is to be sent from subset to another, the leaders handle the message passing
The cost of node sending message to all its neighbors depends on the number of subsets it has neighbors and the distance between the corresponding leaders.

The sparser graph’s radius should be small $O(\log n)$ and density at most linear $O(n)$. We should also be able to construct good sparser division efficiently. Fortunately, this can be done giving total cost of $O(n^2 \log n)$. 
Min-Hop Routing

Consider all links have same unit cost. Now we are counting hops.

Shortest-path spanning tree can now be constructed similar to what we did before, but now all the nodes at minimum distance can be included. This is actually a breadth-first search.

Worst case (line where $s$ at the end) cost is $O(n^2)$. 

Expand more than one level at the leaves in each iteration. Leaves propagate explore message with counter that gets decreased each time new leaf-child is reached. “Explore this many levels”.

Conflicts possible. A new parent with less distance is can be found, in that case discard all work previously done from that node and start from scratch with the new parent.

Cost in sparse networks is only $O(n^{1.5})$, but worst case dense networks cost is still $O(n^2)$. 
Routing is the process of determining path between two entities.

Routing protocols create routing tables of the whole network for each entity.

Different protocols’ costs vary from $O(n^{1.5})$ to $O(n^4)$. 