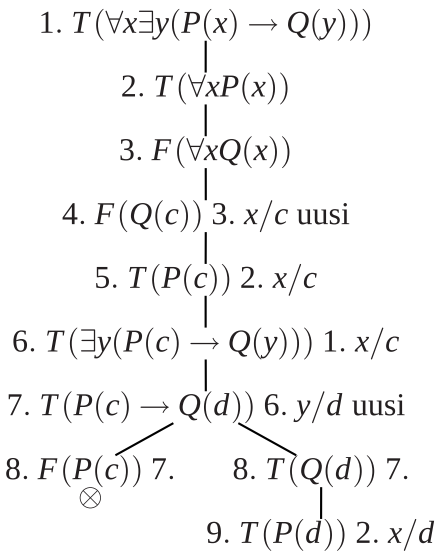
T-79.3001 Logic in Computer Science: Foundations Spring 2009
Exercise 9 ([Nerode and Shore, 1997], Predicate Logic, Chapters 4 and 9)
April 2 – 6, 2009

Solutions to demonstration problems

Solution to Problem 4

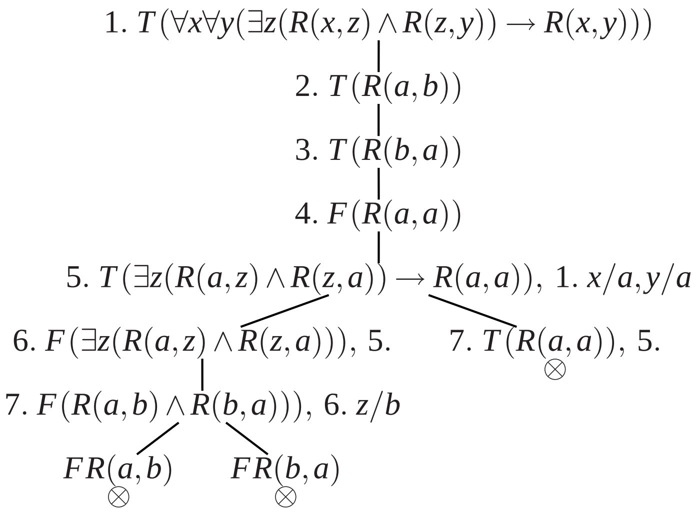
a) Tableau proof:



It seems that the tableau cannot be finished. We read a counter-example $\mathcal{S}$ from an open branch: domain $U = \{1,2\}$, interpretations for constants $c^{\mathcal{S}} = 1$ and $d^{\mathcal{S}} = 2$, and interpretations for predicates $P^{\mathcal{S}} = \{1,2\}$ and $Q^{\mathcal{S}} = \{2\}$.

Since the *tableau is not finished*, we *need to check the counter-example*. Now, we get $\mathcal{S} \models \forall x \exists y (P(x) \to Q(y))$, $\mathcal{S} \models \forall x P(x)$ and $\mathcal{S} \not\models \forall x Q(x)$ for $\mathcal{S}$.

b) Tableau proof:



All the branches in the tableau are contradictory and thus the claim holds.

Solution to Problem 5

We choose the following predicates:

$G(x)$ = "x is guilty",
$L(x)$ = "x is liar",
$A(x)$ = "x is accused", and
$W(x)$ = "x is witness".

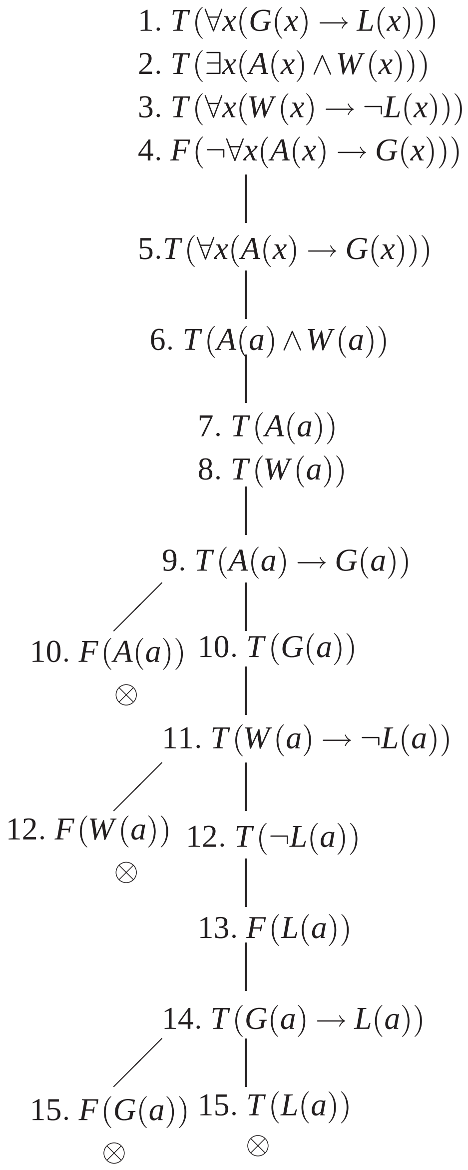
The sentences are:

(i) $\forall x(G(x) \to L(x))$,

(ii) $\exists x(A(x) \wedge W(x))$, ja

(iii) $\forall x(W(x) \to \neg L(x))$.

and we want to show that $\neg \forall x(A(x) \to G(x))$. The tableaux proof is as follows.


1. $T(\forall x(G(x) \to L(x)))$
2. $T(\exists x(A(x) \land W(x)))$
3. $T(\forall x(W(x) \to \neg L(x)))$
4. $F(\neg\forall x(A(x) \to G(x)))$
5. $T(\forall x(A(x) \to G(x)))$
6. $T(A(a) \land W(a))$
7. $T(A(a))$
8. $T(W(a))$
9. $T(A(a) \to G(a))$
10. $F(A(a))$
$\otimes$
10. $T(G(a))$
11. $T(W(a) \to \neg L(a))$
12. $F(W(a))$
$\otimes$
12. $T(\neg L(a))$
13. $F(L(a))$
14. $T(G(a) \to L(a))$
15. $F(G(a))$
$\otimes$
15. $T(L(a))$
$\otimes$

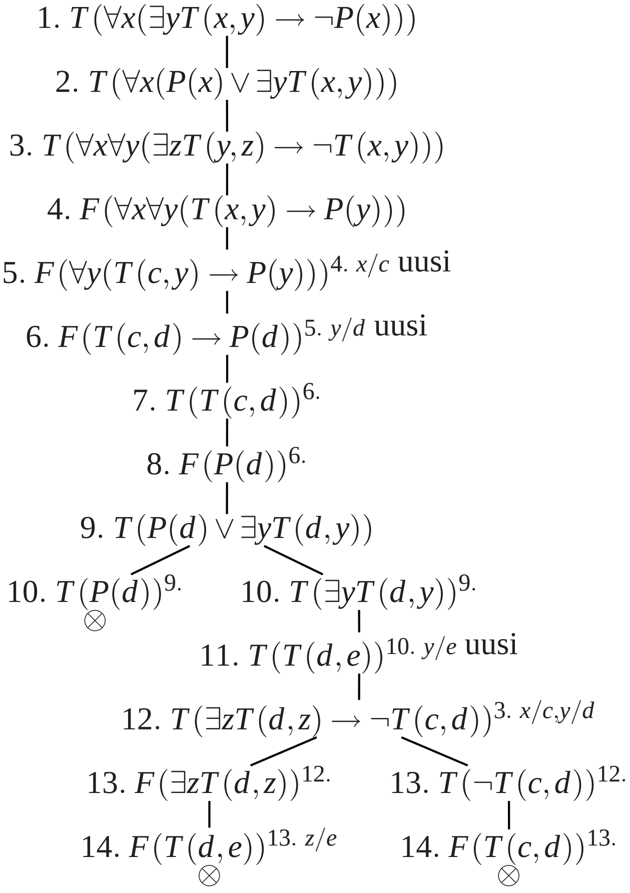
Solution to Problem 6
We use the following predicates:

$T(x,y)$ = “brick x is on brick y”, and
$P(x)$ = “brick x is on the table”.

The set of sentences is:

$$\{\forall x\,(\exists y\,T(x,y) \to \neg P(x)),\ \forall x\,(P(x) \vee \exists y\,T(x,y)),\\ \forall x \forall y\,(\exists z\,T(y,z) \to \neg T(x,y))\}$$

and we want to show that $\forall x \forall y\,(T(x,y) \to P(y))$.
Tableau proof:



$1.\ T(\forall x(\exists y T(x,y) \to \neg P(x)))$

$2.\ T(\forall x(P(x) \vee \exists y T(x,y)))$

$3.\ T(\forall x \forall y(\exists z T(y,z) \to \neg T(x,y)))$

$4.\ F(\forall x \forall y(T(x,y) \to P(y)))$

$5.\ F(\forall y(T(c,y) \to P(y)))^{4.\ x/c}$ uusi

$6.\ F(T(c,d) \to P(d))^{5.\ y/d}$ uusi

$7.\ T(T(c,d))^{6.}$

$8.\ F(P(d))^{6.}$

$9.\ T(P(d) \vee \exists y T(d,y))$

$10.\ T(P(d))^{9.}$ $\otimes$ $\qquad$ $10.\ T(\exists y T(d,y))^{9.}$

$11.\ T(T(d,e))^{10.\ y/e}$ uusi

$12.\ T(\exists z T(d,z) \to \neg T(c,d))^{3.\ x/c, y/d}$

$13.\ F(\exists z T(d,z))^{12.}$ $\qquad$ $13.\ T(\neg T(c,d))^{12.}$

$14.\ F(T(d,e))^{13.\ z/e}$ $\otimes$ $\qquad$ $14.\ F(T(c,d))^{13.}$ $\otimes$

Note: 1) can be equivalently stated as $\forall x \forall y(T(x,y) \to \neg P(x))$ and 3) as $\forall x \forall y \forall z(T(y,z) \to \neg T(x,y))$. How would the tableau look if you used these sentences?